Proceedings of the Finnish Mathematical Days 2016

7.-8.1.2016, Turku

Scientific committee

Tuomas Hytönen	Peter Hästö	Juhani Karhumäki	Antti Kupiainen
Matti Lassas	Kaisa Matomäki	Marko Mäkelä	

Invited plenary speakers

Mats Gyllenberg	Kaisa Matomäki	Eero Saksman	Mikhail Volkov

Invited young researchers

Antti Hannukainen	Joonas Ilmavirta	Tuomas Orponen	Ville Salo

Sessions

Analysis I	Applied I	Discrete I	Stochastic I	Logic I	Inverse I
Analysis II	Applied II	Discrete II	Stochastic II	Logic II	Inverse II
Analysis III	Applied III	Discrete III	Math. phys.	Statistics	Education I
Analysis IV	Applied IV	Discrete IV	Number th.	Systems th.	Education II

Preface

The Finnish Mathematical Days 2016 were held at the University of Turku in January 7.–8.1.2016. This was the third time when the days were in Turku. The days constitute the national survey of education and research of Finnish mathematics. The lectures cover both basic research and applications.

The program of these Mathdays was traditional: There were four plenary lectures given by M. Volkov (Ural Federal University), Eero Saksman (University of Helsinki), Mats Gyllenberg (University of Helsinki) and Kaisa Matomäki (University of Turku). There were also four invited lectures given by young, recently awarded Finnish mathematicians, namely those by Joonas Ilmavirta (University of Jyväskylä), Tuomas Orponen (University of Helsinki), Antti Hannukainen (Aalto University) and Ville Salo (University of Turku).

In addition, the program contained 24 focused sessions presented in six parallel ones. They covered more or less all directions of current Finnish mathematics. Most sessions contained three presentation, chosen by the chairs of the sessions.

Short abstracts of all presentations are collected in these electronic notes. Later they are supposed to be published as TUCS proceedings.

There were two special events in the program. First, the donation of a portrait of the first internationally recognized Finnish mathematician Anders Lexell to the University of Turku. Lexell was a professor of mathematics at the Academy of Turku in the 18th century. A presentation on the scientific impact of Lexell was given by Mauri Valtonen (University of Turku). The other special event was a panel titled as "Nuorten tutkijoiden urakehitys ja haasteet".

Turku, January 5, 2016

Juhani Karhumäki Aleksi Saarela

Contents

1 Invited plenary speakers

Mats Gyllenberg (HY): Rock, scissors, paper – what a children's game can tell us about evolution

It is a wide spread misconception that evolution optimizes some quantity like "fitness" or reproductive success. In this talk I give a brief introduction to adaptive dynamics, which is a mathematical theory that explicitly takes into account the interaction between population dynamics (ecology) and evolution by natural selection. Using the well-known rock-scissors-paper-game as a metaphor, I give necessary and sufficient conditions for when there is a function which is optimized by natural selection. It turns out that evolutionary optimization is extremely rare and hardly can happen in nature.

Kaisa Matomäki (TY): Around the Möbius function

The Möbius function plays a central role in number theory; both the prime number theorem and the Riemann Hypothesis are naturally formulated in terms of the amount of cancellation one gets when summing the Möbius function. In a recent joint work with Maksym Radziwiłł we have shown that the sum of the Möbius function exhibits cancellation in "almost all intervals" of arbitrarily slowly increasing length, i.e. we have shown that, for any $h = h(X) \to \infty$ with $X \to \infty$, one has

$$\sum_{x \leq n < x+h} \mu(n) = o(h)$$

for almost all $x \in [X, 2X]$. This goes beyond what was previously known conditionally on the Riemann Hypothesis.

Our result holds in fact in much greater generality, and has several further applications. For instance the general result implies that, for any $\varepsilon > 0$, there exists $C = C(\varepsilon) > 0$ such that, for all large enough x, the interval $[x, x + C\sqrt{x}]$ contains numbers whose all prime factors are at most x^{ε}. This settles a conjecture on "smooth" numbers and is related to the running time of Lenstra's factoring algorithm.

Eero Saksman (HY): On the Gaussian multiplicative chaos

We try to explain what Gaussian multiplicative chaos is, and recall its basic properties. In addition, we revisit the uniqueness problem in the case of critical chaos, and if time permits, some of the (partly heuristic) connections to e.g. Liouville quantum gravity or the Riemann zeta function will be discussed.

Mikhail Volkov (Ural Federal University): Matrix identities involving multiplication and transposition

Matrices and matrix operations constitute basic tools for algebra, analysis and many other parts of mathematics. Important properties of matrix operations are often expressed in form of laws or identities such as the associative law for multiplication of matrices. Studying matrix identities that involve multiplication and addition is a classic research direction motivated by several important problems in geometry and algebra. Matrix identities involving along with multiplication and addition also certain involution operations (such as taking the usual or symplectic transpose of a matrix) have attracted much attention as well.

If one aims to classify matrix identities of a certain type, then a natural approach is to look for a collection of "basic" identities such that all other identities would follow from these basic identities. Such a collection is usually referred to as a basis. For instance, all identities of matrices over an infinite field involving only multiplication are known to follow from the associative law. Thus, the associative law forms a basis of such "multiplicative" identities. For identities of matrices over a finite field or a field of characteristic 0 involving both multiplication and addition, the powerful results by Kruse–L'vov and Kemer ensure the existence of a finite basis. In contrast, multiplicative identities of matrices over a finite field admit no finite basis.

Here we consider matrix identities involving multiplication and one or two natural one-place operations such as taking various transposes or Moore–Penrose inversion. Our results may be summarized as follows.

None of the following sets of matrix identities admits a finite basis:

- the identities of $n \times n$-matrices over a finite field involving multiplication and usual transposition;
- the identities of $2n \times 2n$-matrices over a finite field involving multiplication and symplectic transposition;
- the identities of 2×2-matrices over the field of complex numbers involving either multiplication and Moore–Penrose inversion or multiplication, Moore–Penrose inversion and Hermitian conjugation;
- the identities of Boolean $n \times n$-matrices involving multiplication and transposition.

2 Invited young researchers

Antti Hannukainen (AYO): On numerical solution of eigenvalues and eigenfunctions

Computing eigenvalues and eigenfunctions of differential operators is of great interest in many fields of engineering and physics. For example, when buildings

are designed, eigenvalues are computed as a part of the design process to eliminate unwanted natural vibration frequencies.

Eigenvalues and eigenfunctions of symmetric differential operators are typically computed by approximating the eigenproblem in a finite dimensional space. The resulting approximate problem can be formulated as a matrix equation and solved by using a suitable method, e.g., Lanzos or QR - iteration. This strategy leads to a set of approximate eigenvalues and eigenfunctions, whose accuracy should be estimated. One has to control two error sources, the error related to the selection of the finite dimensional space and the error generated by the eigensolver.

In this talk, I discuss using finite element method to construct the finite dimensional space and suitable numerical methods for solving the resulting finite dimensional eigenproblem. In addition, I present a prior convergence estimates for the difference between the exact and the approximate eigenvalues and the gap between exact and approximate eigenspaces. The topic is classical and the form of the convergence results is well known. The novelty of the presented approach is in a new technique used to derive the estimates.

Joonas Ilmavirta (JY):
From electrical measurements to tomography on groups

We start from a practical problem related to indirect electrical measurements and formulate it mathematically. In a special case this leads us to the broken ray transform. Studying this transform on a square leads to X-ray tomography on tori. The methods that work on tori extend to compact Lie groups and also finite groups. A journey starting with a practical problem takes us through a variety of mathematical fields including partial differential equations, inverse problems, Fourier analysis, Lie groups, representation theory, Frobenius groups and geodesics on finite groups.

Tuomas Orponen (HY):
Distances of AD-regular sets

The "distance set problems" of Erdös and Falconer ask to determine the relationship between the size of a planar set K, and that of its distance set $D(K) = \{|x - y| : x, y \in K\}$. The Erdös problem (for finite sets) was essentially solved by Guth and Katz in 2010, but Falconer's version (for sets of positive Hausdorff dimension) remains open. I discuss what we know so far.

Ville Salo (TY):
Reversible cellular automata and the automorphism group of a full shift

Cellular automata are functions that act on bi-infinite words, where the operation at each individual position (or "cell") of the word is given by the same local rule. The reversible cellular automata form the topological automorphism group of the

full shift topological dynamical system (the dynamical system of infinite words). We review (and slightly extend) some of the classical literature on this group G, with particular emphasis on groups embeddable in G, and discuss closure properties of its set of subgroups. We also cover some known results and open problems about the group G itself.

3 Analysis I, complex and hypercomplex analysis with applications

Organized by: Sirkka-Liisa Eriksson, Heikki Orelma (TTY).

Vesa Vuojamo (TTY): Hyperbolic function theory

We study the generalization of complex analysis into higher dimensions. We use the Clifford algebra as a means to introduce monogenic functions which are the generalizations of analytic functions.

Let V be a (non-degenerate) bilinear space of signature (p, q). The Clifford algebra $\mathbb{R}_{p,q}$ can be realized as a quotient of the tensor algebra over V by the ideal generated by

$$x \otimes x - \langle x, x \rangle \quad x \in V.$$

Given an orthonormal basis this means that

$$\begin{array}{ll} e_i^2 = 1 & i = 1, \ldots, p \\ e_i^2 = -1 & i = p+1, \ldots, p+q \\ e_i e_j = -e_j e_i & i \neq j. \end{array}$$

Now set $p = 0$ and $q = n$. The monogenic functions can be defined as the null solutions of the Cauchy-Riemann operator

$$D_x = \partial_{x_0} + \sum_{i=1}^{n} e_i \partial_{x_i}.$$

This operator also factorizes the standard Laplace-operator like

$$\Delta = \overline{D}_x D_x = D_x \overline{D}_x.$$

This is one way to generalize analytic functions but not everything is preserved from classical complex analysis. The first thing to note is that powers

$$x^k = (x_0 + x_1 e_1 + \cdots + x_n e_n)^k \quad k \in \mathbb{Z}$$

are not monogenic.

We take the point of view that powers should be included in our generalized analytic functions. We can achieve this by introducing a modification of the Cauchy-Riemann operator

$$M = D_x + \frac{n-1}{x_n} Q'$$

where $(\cdot)'$ is an involution which acts like $e_i' = -e_i$ and Q is a mapping which gives the coefficient of the e_n-component and we denote $Q'(x) = Q(x')$. We introduce the function theory based on this modified operator.

Terhi Kaarakka (TTY): Spectral density function of the Doob transformation of fractional Brownian motion (i.e. the fractional Ornstein-Uhlenbeck process)

Bochner proved a connection, that every non-negative definite function can be represented as a Fourier transformation or Riemann–Stieltjes integral, where the integrator is an odd increasing function Δ.

It is possible to do Lebesgue decomposition of the measure induced by Δ denoted by $d\Delta(\gamma)$, as follows

$$d\Delta(\gamma) = d\Delta^\circ(\gamma) + \Delta'(\gamma)d\gamma.$$

The singular part $d\Delta^\circ(\gamma)$ is singular with respect to Lebesgue measure $d\gamma$ and *the nonsingular part*, which is *absolutely continuous* with respect to the Lebesgue measure $d\gamma$.

We have a Riemann-Stieltjes measure Δ, which can be thought of as a Borel measure. Every Borel measure can be decomposed using the Lebesgue decomposition theory as before. Hence

$$\int \mathrm{e}^{i\gamma t} d\Delta(\gamma) = \int \mathrm{e}^{i\gamma t}\Delta'(\gamma)d\gamma + \int \mathrm{e}^{i\gamma t} d\Delta^\circ(\gamma).$$

Since every stationary Gaussian process has a non-negative definite covariance function of one variable, this theorem may be applied to find an odd increasing function Δ' that is called *the spectral density function of the underlying Gaussian process.*

There is a trigonometric isometry between the Gaussian Hilbert space G (that is Hilbert space formed by Gaussian processes) and $L^2(\mathbb{R}, d\Delta)$ via the formula

$$\mathbb{E}(X_{t_1}X_{t_2}) = \int \mathrm{e}^{i\gamma t_1}\overline{\mathrm{e}^{i\gamma t_2}} d\Delta(\gamma), \tag{1}$$

for all $t_1, t_2 \in \mathbb{R}$. This is the main tool of the prediction, since it connects the two spaces and their elements. Dym and McKean described a prediction method and Lamperti also studied the same problem.

The Ornstein–Uhlenbeck diffusion can be constructed from Brownian motion via a Doob transformation and also from a solution of the Langevin stochastic differential equation. Both of these processes have the same finite dimensional distributions. We can define two different fractional Ornstein–Uhlenbeck processes similar ways as Ornstein–Uhlenbeck processes, but using fractional Brownian motion instead of Brownian motion. However the solution of the Langevin stochastic differential equation, which driving process is fractional Brownian motion and

a Doob transformation of fractional Brownian motion do not have same finite dimensional distributions, but both of them are stationary. In this presentation we only use the Doob transformation of fractional Ornstein–Uhlenbeck process and therefore we only define that.

Definition 1. *Let $Z = \{Z_t : t \geq 0\}$ be the fractional Brownian motion. Then the process $X^{(D,\alpha)} = \{X_t^{(D,\alpha)} : t \in \mathbb{R}\}$ is* the Doob transformation of fBm *(abbreviated fOU) if*

$$X_t^{(D,\alpha)} := \mathrm{e}^{-\alpha t} Z_{\tau_t},$$

where $t \in \mathbb{R}$, $H \in (0,1)$, $\alpha > 0$ and $\tau_t = \dfrac{He^{\frac{\alpha}{H}t}}{\alpha}$.

The main idea of this presentation is to show that

Theorem 2. *The spectral density function of the Doob transformation of fBm (fOU) is*

$$\Delta'(\gamma) = \frac{1}{2}\left(\frac{H}{\alpha}\right)^{2H}\left(\frac{\alpha}{\pi}\frac{1}{\gamma^2+\alpha^2} - \frac{\alpha}{\pi}\sum_{n=1}^{\infty}\binom{2H}{n}\frac{(-1)^n\left(\frac{n}{H}-1\right)}{\gamma^2+\left(\frac{\alpha n}{H}-\alpha\right)^2}\right).$$

Heikki Orelma (TTY): Usean muuttujan superanalyysistä

Superanaalyysiksi kutsutaan venäläinen matemaatikon, Felix Berezin esittämää analyysin teoriaa, missä tarkastellaan funktiota, jotka riippuvat ”tavallisten” (bosonisten) muuttujien x_j lisäksi antikommutoivista (fermionisista) muuttujista $x_j^{\backprime}$. Muuttujat toteuttavat siis laskusäännöt

$$\begin{aligned} x_i x_j &= x_j x_i, \\ x_i^{\backprime} x_j^{\backprime} &= -x_j^{\backprime} x_i^{\backprime}, \\ x_i x_j^{\backprime} &= x_j^{\backprime} x_i. \end{aligned}$$

Berezinin teorian laajennuksen vektorimuuttujan funktioille esitteli vuosituhannen vaihteessa Frank Sommen. Hänen ideansa on tarkastella vektorimuuttujien

$$x = \mathbf{x} + \mathbf{x}^{\backprime},$$

funktioita, missä

$$\mathbf{x} = x_1 e_1 + \cdots + x_m e_m$$

ja

$$\mathbf{x}^{\backprime} = x_1^{\backprime} e_1^{\backprime} + \cdots + x_{2n}^{\backprime} e_{2n}^{\backprime}.$$

Lähtökohtana tässä näkökulmassa on tarkastella reaalista assosiatiivista algebraa $\mathcal{P}_{m,2n} = \mathrm{Alg}\{x_i, e_i, x_j^{\backprime}, e_j^{\backprime}\}$, kun $i = 1, ..., m$ ja $j = 1, ..., 2n$, missä vektorimuuttujat

toteuttavat seuraavat laskusäännöt:

$$\begin{aligned} e_j e_k + e_k e_j &= -2\delta_{ij}, \\ e_i e_j^{\backprime} &= -e_j^{\backprime} e_i, \\ e_{2j}^{\backprime} e_{2k}^{\backprime} &= e_{2k}^{\backprime} e_{2j}^{\backprime}, \\ e_{2j-1}^{\backprime} e_{2k-1}^{\backprime} &= e_{2k-1}^{\backprime} e_{2j-1}^{\backprime}, \\ e_{2j-1}^{\backprime} e_{2k}^{\backprime} - e_{2k}^{\backprime} e_{2j-1}^{\backprime} &= \delta_{ij}. \end{aligned}$$

Huomaamme, että vektorit e_j generoivat Cliffordin algebran ja $e_j^{\backprime}$ Weylin algebran. Esitelmässä tarkastellaan Diracin yhtälön ratkaisemista tästä näkökulmasta.

4 Analysis II, functional analysis

Organized by: Mikael Lindström (ÅA).

Pekka Salmi (OY): Idempotent states

An idempotent state on a locally compact group is a probability measure that is an idempotent with respect to convolution, i.e. $\mu * \mu = \mu$. The Kawada–Itô theorem characterises such idempotent states as the normalised Haar measures of compact subgroups. Hence for example the idempotent states on the circle group $\mathbb{T}$ consisting of all unimodular complex numbers are the normalised one-dimensional Lebesgue measure on $\mathbb{T}$ and the normalised counting measures of the finite subgroups of $\mathbb{T}$. On the dual side, idempotents states on group C*-algebras are characteristic functions of open subgroups. In this talk we consider idempotent states on coamenable locally compact *quantum* groups and discuss connections between idempotent states, quantum subgroups and invariant C*-subalgebras. The talk is based on joint work with Adam Skalski.

Pekka Nieminen (HY): Rigidity of composition operators on Hardy spaces

We consider rigidity properties related to the strict singularity of composition operators $C_\varphi : f \mapsto f \circ \varphi$ on the analytic Hardy spaces H^p for $p \neq 2$, where $\varphi : \mathbf{D} \to \mathbf{D}$ is a given analytic map and $\mathbf{D}$ is the unit disc in the complex plane. We show that for $1 < p < \infty$, $p \neq 2$, the operator C_φ is either compact or else fixes a copy of the sequence space ℓ^p in H^p. This is joint work with Jussi Laitila and Hans-Olav Tylli (Helsinki).

Jarno Talponen (ISY): On long unconditional basic sequences in Banach spaces

The talk is about finding long unconditional basic sequences in Banach spaces of high density. In general, it is a natural question to ask if a mathematical structure

which is suitably large must contain a small substructure with some given restrictive nice properties. By now such considerations and also the application of infinitary combinatorics in analyzing different aspects of Banach spaces are classical topics. We begin the talk with some implications of Ramsey theoretic principles in logic (namely partition relations) to the geometry of Banach spaces. We briefly discuss topological games in Banach spaces. Then we take a look at some (apparently) simple geometric properties of the spaces which partially replace combinatorial principles in the arguments.

5 Analysis III, geometric measure theory

Organized by: Antti Käenmäki (JY).

Ville Suomala (OY): Arithmetic combinatorics on fractals

For sets $A \subset \mathbb{N}$ a fundamental problem and one of the origins of arithmetic combinatorics is to find conditions which guarantee that A contains certain point configurations like 3-term arithmetic progressions. Recall that $\{x_1, \ldots, x_m\}$ is called an arithmetic progression if $x_{i+1} - 2x_i + x_{i-1} = 0$ for all $1 < i < m$. The classical Roth's theorem on arithmetic progressions implies that the presence of 3-term arithmetic progression in A is guaranteed via the positivity of the upper density

$$\delta(A) = \limsup_{n\to\infty} \frac{|\{1 \leq k \leq n \,:\, k \in A\}|}{n}\,.$$

Szeméredi's theorem generalizes this for arithmetic progressions of arbitrary length $m \geq 4$ and the celebrated Green-Tao theorem, which has stimulated a lot of research over the past decade or so, provides an analogue for subsets of the primes.

For sets $A \subset \mathbb{R}$, there is no obvious replacement of the upper density but one might still hope that "large subsets" of $\mathbb{R}$ contain arithmetic progressions. Indeed, a well known corollary of the Lebesgue density theorem implies that a set with positive Lebesgue measure contains arbitrary long arithmetic progressions, and more generally, similar copies of any finite set.

For sets of zero Lebesgue measure, various different notions of fractal dimensions may be used to measure their size. However, a large value of dimension alone cannot be used to conlcude the existence of arithmetic progressions. For instance, there are sets $A \subset \mathbb{R}$ of full Hausdorff dimension without any arithmetic progressions (Keleti 1998). Heuristics from the discrete analogues suggests that a more natural concept in this context might be the Fourier dimension which measures the decay of the Fourier transform of measures supported in A. There are some positive results in this direction showing that appropriate dimensionality and Fourier decay conditions on A imply the existence of 3-term arithmetic progressions and more general geometric configurations in A (e.g. Łaba and Pramanik 2009, Chan, Łaba, and Pramanik 2013, Henriot, Łaba, and Pramanik 2015). However, the decay of

the Fourier transform has to be very uniform and it is not enough to assume that the Fourier dimension of A is large (Shmerkin 2015).

In this talk, we take another perspective and discuss the existence of arithmetic progressions and various other geometric configurations for sets $A \subset \mathbb{R}$ as well as $A \subset \mathbb{R}^d$ obtained as limits of random constructions. In particular, we examine the situation for fractal percolation which is defined by the following branching process: Given $d \in \mathbb{N}$ and a parameter $0 < p < 1$, we subdivide the unit cube in $\mathbb{R}^d$ into 2^d equal sub-cubes. We retain each of them with probability p and discard it with probability $1 - p$, with all the choices independent. For each of the retained cubes, we continue inductively in the same fashion, by further subdividing them into 2^d equal sub-cubes, retaining them with probability p and discarding them otherwise, with all the choices independent. The fractal percolation limit set $A = A^{\mathrm{perc}(d,p)} \subset [0,1]^d$ is the set of points which are kept at each stage of the construction. It is well known that A is nonempty with positive probability if and only if $p > 2^{-d}$. Furthermore, its dimension is given by the formula

$$\dim A = s(d,p) = d + \log_2 p\,,$$

almost surely on $A \neq \varnothing$.

Our main results characterise the presence of various finite configurations in $A^{\mathrm{perc}(d,p)}$ in terms of the almost sure dimension $s = s(d,p)$ (or equivalently, in terms of the parameter p). For instance, if $m \in \mathbb{N}$ and $s = s(1,p) > 1 - 2/m$, then $A^{\mathrm{perc}(1,p)} \subset \mathbb{R}$ contains m-term arithmetic progressions and more generally, similar copies of any m-element set $\{a_1, \ldots, a_m\} \subset \mathbb{R}$ almost surely on $A \neq \varnothing$. We will also provide various generalizations and variants in the case $d \geq 2$. For example, if $s(d,p) > 2/3$, we show that almost surely on $A \neq \varnothing$, the angles formed by the triples in A constitute to $]0, \pi[$. Furthermore, we will examine the size and structure of the sumsets $S_1 A + \ldots + S_m A$ for $S_i \in GL_d(\mathbb{R})$.

This talk is based on joint work with Pablo Shmerkin (Torcuato di Tella University, Buenos Aires).

Tapio Rajala (JY):
Tangents of metric measure spaces

I will review some results on Gromov-Hausdorff tangents of metric spaces. I will start with questions of the type: What is the relation between metric dimensions (such as Hausdorff, Assouad or Nagata dimension) of the space and its tangents? What can we say about the space if its tangents are everywhere unique or if we converge towards the tangents uniformly? Some answers to these questions are from my joint work with Enrico Le Donne [5].

I will continue with tangents of Lipschitz differentiability spaces and with tangents of spaces with Ricci curvature lower bounds. Here I will concentrate on my paper with Nicola Gigli and Andrea Mondino [4] and on my preprint with Fabio Cavalletti [1]. Closely related to these results are the splitting theorem of Gigli [3], the rectifiability result of Mondino and Naber [6] and the studies on local structure of Lipschitz differentiability spaces by Cheeger, Kleiner and Schioppa [2] and Schioppa [7].

References

[1] F. Cavalletti and T. Rajala, *Tangent lines and Lipschitz differentiability spaces*, Preprint, arXiv:1503.01020, (2015).

[2] J. Cheeger, B. Kleiner and A. Schioppa, *Infinitesimal structure of differentiability spaces, and metric differentiation*, Preprint, arXiv:1503.07348, (2015).

[3] N. Gigli, *The splitting theorem in non-smooth context*, Preprint, arXiv:1302.5555, (2013).

[4] N. Gigli, A. Mondino and T. Rajala, *Euclidean spaces as weak tangents of metric measure spaces with Riemannian Ricci curvature bounded below*, J. Reine Angew. Math. **705** (2015), 233–244.

[5] E. Le Donne and T. Rajala, *Assouad dimension, Nagata dimension, and uniformly close metric tangents*, Indiana Univ. Math. J. **64** (2015), no. 1, 21–54.

[6] A. Mondino and A. Naber, *Structure theory of metric-measure spaces with lower Ricci curvature bounds I*, Preprint, arXiv:1405.2222 (2015)

[7] A. Schioppa, *The Lip-lip equality is stable under blow-up*, Preprint, arXiv:1505.06363, (2015).

Eino Rossi (JY):
Local structure of fractal sets

One common definition of a fractal is that it consists of smaller copies of itself. This indicates that when zooming into a fractal set one could expect to see the same set over and over again. In some sense this is true, but even for self-similar sets this is not the whole truth. In this talk, I try to give a more detailed study of what can we actually see when looking deeper and deeper into a fractal set. The main focus is on the geometric limit objects, such as tangent sets and micro sets, of this zooming in process.

6 Analysis IV, geometric analysis

Organized by: Changyu Guo (JY), Istvan Prause (HY).

Katrin Fässler (JY):
Uniqueness of minimizers for a distortion functional in the Heisenberg group

Quasiconformal maps have been considered first in the complex plane as maps that are 'as conformal as possible', that is, of smallest possible distortion within a given class of mappings. Here, 'distortion' is understood as L^∞-norm of a suitable

pointwise distortion function, but one might as well study minimizers of other distortion functionals within the class of quasiconformal maps. In this spirit, P.P. Belsinkii classified the minimizers of a certain weighted average distortion among quasiconformal maps between two annuli in the plane.

In this talk I will discuss an analogous problem in a non-Euclidean metric space – the sub-Riemannian Heisenberg group – and I will show that in contrast to the situation considered by Belsinkii, the problem has a unique solution in the Heisenberg group. The argument illustrates some of the characteristic features that govern the geometry in the Heisenberg group. This is joint work with Z. Balogh and I. Platis.

Oleg Ivrii (HY):
On Makarov's principle in conformal mapping

We examine several characteristics of conformal maps that resemble the variance of a Gaussian: (1) asymptotic variance, (2) the constant in Makarov's law of iterated logarithm and (3) the second derivative of the integral means spectrum at the origin, amongst others. While these quantities need not be equal in general, they agree for domains whose boundaries are regular fractals such as Julia sets or limit sets of quasi-Fuchsian groups. We show these characteristics have the same universal bounds over various collections of conformal maps. As an application, we show that the maximal Hausdorff dimension of a k-quasicircle is strictly less than $1 + k^2$. (Part of this work is joint with I. Kayumov.)

Luděk Kleprlík (JY):
Composition operators on W^1X and quasiconformal mappings

Let $\Omega_1, \Omega_2 \subset \mathbb{R}^n$. We study the optimal conditions on a homeomorphism $f : \Omega_1$ onto Ω_2 which guarantee that the composition $u \circ f$ belongs to the Sobolev space $W^1X(\Omega_1)$ for every $u \in W^1X(\Omega_2)$. First we discuss the results for classical Sobolev spaces, Sobolev-Orlicz spaces and Sobolev-Lorentz spaces. Then we show general theorem which give us that if rearrangement invariant space X satisfies some geometry property ($\|\chi_{B(x,st)}\|_X \approx s^{\frac{n}{q}} \|\chi_{B(x,t)}\|_X$) then f is q-quasiconformal, i.e. $|D(f(x))|^q \leq K|J_f(x)|$ for a.e. $x \in \Omega$.

7 Applied mathematics I, numerical analysis

Organized by: Rolf Stenberg (AYO).

Jukka Tuomela (ISY):
On the numerical solution of chromatographic separation models

In a chromatographic separation model one has several species moving in a fluid and interacting with each other. For each species there are 3 different variables: concentration of the liquid phase C, concentration of the solid phase S and equilibrium concentration in the interface between 2 phases E. In addition we have the vector field describing the fluid flow but this is supposed to be known.

If we have k species the model has k convection diffusion equations for C, k ODE for S and finally k constraints or algebraic equations relating C and E. The numerical difficulty of solving this kind of system is due to the fact that the algebraic equations are highly nonlinear. The standard approach has been to design special numerical schemes which would preserve the constraints as well as possible. Here we take a different approach. We differentiate the constraint equations to obtain genuine dynamics also for E variables. However, we do not forget the algebraic constraints and treat the whole system as an overdetermined problem.

One might say that in the standard approach one tries to keep the system simple and one has to design complicated numerical schemes while in our approach we make the system a bit more complicated but then we can use any standard methods for PDE. The advantage is thus that this methodology is not dependent on the particular form of the system.

We illustrate our approach using a 2 species model where one species is the salt and the other a certain protein. In this case one can actually visualize the constraint manifold as a surface in $\mathbb{R}^3$. Nonlinearities are indeed very strong but the manifold itself is not geometrically complicated. Hence there is no reason why the problem should be intrinsically numerically complicated. Indeed our computations show that with our modified model one obtains good results with standard numerical methods.

Lauri Perkkiö (AYO):
Energy balanced numerical integration for dissipative DAE – Application in electric machine simulations

When simulating magnetic and electric fields in an electric machine, after a standard finite element discretization one arrives at a non-linear equation of form

$$M\dot{u} = f_1(u, v) + g(t) \tag{2}$$

$$0 = f_2(u, v), \tag{3}$$

where the coefficients u, v of the finite element basis functions are to be solved. Equations (3) result from one material parameter, the electrical conductivity σ, being zero in certain subdomains. This kind of problem is called an *differential-algebraic equation* (DAE), as it involves an ordinary differential equation (ODE) coupled to an "algebraic equation" ("algebraic" does not refer to any polynomials

or such, but to an equation without time derivative). This is an index-1 DAE, the simplest type, because one can in principle solve $v = F(u)$ from (3). However, in practice this is impossible due non-linearity and large number of unknowns.

The conventional time integration methods for the system (2-3) are Implicit Euler (Backward Euler) and Crank-Nicolson (Trapezoid) methods. Physically, the sum of powers (as in Watts) should be zero,

$$P_{bal} = \frac{d}{dt}W_{stored} + P_{loss} + P_{work} + P_{input} = 0, \tag{4}$$

but it is well-known that using one of the conventional methods and then (at post-processing) computing the previous terms one does not get exactly $P_{bal} = 0$, even if a linear problem is studied.

Our aim is to construct an integration scheme that satisfies $P_{bal} = 0$ exactly, at least when a linear and non-moving ($P_{work} = 0$) problem is studied. The motivation to our work comes from solving the term P_{work}; there are several methods for computing it from the finite element field solution, but the results are not consistent. However, as an alternative method to compute P_{work}, it is proposed that if all the other terms in (4) were known accurately then P_{work} is also known accurately, assuming that the numerical balance P_{bal} is small enough.

The proposed scheme is a slight modification of so called collocation method (a subfamily of Implicit Runge-Kutta methods). Shortly, in each time step the unknowns u and v are considered to be n-th order polynomials, $p(t) = (u(t), v(t)), t \in [t_i, t_i + h]$. The coefficients of p are solved such that the ODE (2) is required to hold at the Gaussian quadrature points, and the AE (3) is required at the endpoint $t_i + h$, and if $n > 1$, at as weighted sum of the Gaussian quadrature points. Thus, the simplest of these methods reads just as "use midpoint method for ODE, but require AE at the endpoint". These methods give the maximal accuracy for given n, and in addition numerical tests show that they work properly with the given DAE-system. It can be shown that the methods give exactly $P_{bal} = 0$ for a linear, non-moving problem. For non-linear problems, numerical tests show that the error for P_{bal} is smaller compared to the conventional methods, even if the computational workload and numerical error are of the same order.

Tom Gustafsson (AYO): Numerical solution of the obstacle problem using mixed and stabilized finite element methods

Joint work with Rolf Stenberg and Juha Videman.

The obstacle problem is a prototypical constrained minimization problem related, e.g., to the deformation of an elastic membrane constrained by a rigid obstacle [1]. Let $\Omega \subset \mathbb{R}^2$ be a sufficiently smooth domain. Furthermore, let $f \in L^2(\Omega)$ be a force field acting on the membrane and $g \in \{w \in H^1(\Omega) \cap C(\overline{\Omega}) : w|_{\partial\Omega} \leq 0\}$ be a rigid obstacle. Then we seek for a function $u : \Omega \to \mathbb{R}$ that solves the constrained minimization problem

$$\inf_{u \in H_0^1(\Omega)} \frac{1}{2} \int_\Omega |\nabla u|^2 \, \mathrm{d}x - \int_\Omega f u \, \mathrm{d}x \quad \text{s.t.} \quad u \geq g. \tag{5}$$

Defining $K = \{w \in H^1(\Omega) : w \geq g\}$ and $\Lambda = \{\mu \in H^{-1}(\Omega) : \langle \mu, v \rangle \geq 0 \ \forall v \in K\}$ we can equivalently seek for the pair (u, λ) satisfying the saddle point problem

$$\inf_{u \in H_0^1(\Omega)} \sup_{\lambda \in \Lambda} \frac{1}{2} \int_\Omega |\nabla u|^2 \, \mathrm{d}x - \int_\Omega f u \, \mathrm{d}x - \langle \lambda, u - g \rangle, \tag{6}$$

where $\langle \cdot, \cdot \rangle : H_0^1(\Omega) \times H^{-1}(\Omega) \to \mathbb{R}$ denotes the duality pairing between $H_0^1(\Omega)$ and $H^{-1}(\Omega)$. Following e.g. Ekeland–Temam [2], the optimality system for (6) is

$$\begin{gathered} \int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x - \langle \lambda, v \rangle = \int_\Omega f v \, \mathrm{d}x \quad \forall v \in H_0^1(\Omega), \\ \langle u, \mu - \lambda \rangle \geq \langle g, \mu - \lambda \rangle \ \ \forall \mu \in \Lambda. \end{gathered} \tag{7}$$

We study the discretization of the system (7) by the finite element method. Stability, a priori and a posteriori estimates are discussed and the methods are numerically verified.

References

[1] D. Kinderlehrer and G. Stampacchia, *An Introduction to Variational Inequalities and Their Applications*, Academic Press, 1980.

[2] I. Ekeland and R. Temam, *Analyse Convexe et Problemés Variationnels*, Gauthier-Villars, 1974.

8 Applied mathematics II, numeerinen analyysi

Organized by: Olavi Nevanlinna (AYO).

Tri Quach (AYO): Conjugate function method and conformal mappings in multiply connected domains

Joint work with Harri Hakula and Antti Rasila.

Numerical conformal mappings play an important role in certain engineering applications such as aerodynamics and study of magnetic fields, see e.g. [1, 5, 6]. The purpose of this talk is to extend the idea of Conjugate Function Method presented in [2] for multiply connected domains. The method is based on the harmonic conjugate function and a natural generalisation of a modulus of quadrilaterals given in [3]. The method relies on solving numerically the Laplace equation with Dirichlet-Neumann mixed boundary conditions which is done by using a *hp*-FEM presented in [4]. Several numerical examples of the algorithm are given.

References

[1] T.A. Driscoll and L.N. Trefethen, *Schwarz-Christoffel Mapping*. Cambridge Monographs on Applied and Computational Mathematics, 8. Cambridge University Press, 2002.

[2] H. Hakula, T. Quach, and A. Rasila, *Conjugate Function Method for Numerical Conformal Mappings*, Journal of Computational and Applied Mathematics 237 (2012), 340–353.

[3] H. Hakula, T. Quach, and A. Rasila, *Conjugate Function Method and Conformal Mappings in Multiply Connected Domains*, arXiv:1502.02047.

[4] H. Hakula, A. Rasila, and M. Vuorinen, *On moduli of rings and quadrilaterals: algorithms and experiments*. SIAM J. Sci. Comput. 33(1) (2011), 279–309.

[5] N. Papamichael and N.S. Stylianopoulos, *Numerical Conformal Mapping: Domain Decomposition and the Mapping of Quadrilaterals*, World Scientific Publishing Company, 2010.

[6] R. Schinzinger and P.A.A. Laura, *Conformal mapping: methods and applications*, Revised edition of the 1991 original. Dover Publications, Inc., Mineola, NY, 2003.

Vesa Kaarnioja (AYO): Stochastic modulus of a quadrilateral as a benchmark problem for uncertain domains

Joint work with Harri Hakula.

We consider the stochastic version of the classical capacity problem for uncertain domains. For this problem, there exists a natural error estimate for any realization of any random boundary. This so-called reciprocal error estimate can be used in the error analysis of statistical quantities related to the numerical solution of the model problem by using a sparse grid stochastic collocation method, where the error estimate is applied to each individual solution within the non-intrusive process.

Olli Mali (JY): Worst case scenario approach for convex minimization problems with uncertain data

This talk concerns uncertain data in mathematical models generated by convex minimization problems. Problems are considered in the mixed setting and the fundamental error quantity associated to any conforming approximation (primal-dual pair) is the respective duality gap. The duality gap can be decomposed into two parts. One part is associated with the inaccuracy of the approximation and the

other part shows the inaccuracy resulting from the incompletely known data. The approach is not stochastic. Instead, the effects of the uncertain data are examined in the worst case scenario setting. The proposed methodology is applied to a nonlinear reaction diffusion problem generated by a power growth functional. The respective error quantities or their computable two-sided bounds are presented. Numerical examples demonstrate how they can be used to detect the the accuracy limit, i.e., the limit beyond which additional (numerical or analytical) efforts to improve the approximation make no sense due to the incompletely known data.

9 Applied mathematics III, biomathematics

Organized by: Kalle Parvinen, Laura Elo-Uhlgren, Tero Aittokallio (TY).

Stefan Geritz (HY): The effect of noise and delays on the behavior of a population and the need to be specific about the underlying mechanisms

A population is an ensemble of interacting individuals. Changes in the size and structure of a population are ultimately a consequence of the behavior of the individuals. The map from individual behavior to population behavior typically is many-to-one, i.e., different sets of individual behavior give rise to the same population model. Yet, even then the incorporation of environmental noise or developmental delays into the system can have widely different effects on the population level depending on the particular underlying set of individual behavior. This is illustrated by means of a concrete example.

Leo Lahti: Stochastic processes in microbial population dynamics

A myriad of microbes inhabit the human body with fundamental implications to our physiology and health. The species composition in these microbial ecosystems can rapidly adapt to changes, and is affected by various factors including environmental parameters, species interactions, and stochastic fluctuations. Microbial community profiling studies are providing time series with increasing density and coverage. The analyses have revealed periodic, chaotic, and stochastic patterns that are all contributing to the overall community assembly. Better understanding of the mechanisms determining microbial community stability and dynamics is essential for developing microbiome-based diagnostics and treatments.

The relative contributions of the deterministic and stochastic patterns in microbial community assembly can vary in time and across communities. Hubbell's Unified Neutral Theory of Biodiversity (UNTB) provides a null model for stochastic assembly, based on the hypothesis that all individuals, regardless of their species, are equivalent in terms of fitness. According to this model, the community evolves

following random birth-death processes. Changes in species composition in many natural ecosystems has been found to closely follow the neutral model.

Quantifying how closely variations in community assembly follow the neutral null model can help to understand relative contributions of neutral stochastic drift and other processes driving the overall community assembly. In the UNTB model, the dynamics of species composition within each local community follows neutral birth-death processes and occasional immigration of new individuals from the metacommunity. We assume that whereas the number of distinct individuals and species within each local community is limited at any given time step, the number of distinct species in the metacommunity is hypothetically unlimited. Parsimonious solutions with fewer species are preferred in practice, however; the fundamental biodiversity parameter sets a prior and a soft constraint on the potential number of unique species in the system. This formulation expands former models based on simplified assumptions such as limited species pool, a single community, or equivalent immigration rates across communities.

Computationally tractable and accurate algorithms to fit the full multi-site UNTB with large numbers of individuals, community-specific immigration rates and a potentially unlimited species pool have been missing. We have recently demonstrated that at the limit of large sample sizes, this ecologically motivated formulation converges to the Hierarchical Dirichlet process, an extensively studied model from statistical machine learning. We used this limit as an approximation to develop an efficient Bayesian fitting strategy and an open source implementation for the full multi-site model.

We applied this fitting strategy to analyze a classical example of tropical trees as well as microbial community assembly in the human gut. The tropical tree data set was consistent with the UNTB model but in the human gut, neutral community assembly matched the observations only at more refined taxonomic levels of microbial species. This suggests that microbial community assembly in the human gut is more strongly niche constrained than macroscopic ecosystems; whereas different taxonomic groups can adopt different functional roles, the diversity within such groups may be maintained by neutrality. Moreover, we found that different parameters of the model are correlated with host phenotype and health, including a negative association between overweight and immigration rates within the family Ruminococcaceae. We will also consider implications of stochastic community variation for the theory of alternative stable states, where small perturbations can lead to abrupt catastrophic shifts in ecosystem composition.

Jing Tang (HY):
Mathematical modeling for the rational selection of personalized cancer drug combinations

Making cancer treatment more personalized and effective is one of the grand challenges in our health care system. However, many drugs have entered clinical trials but so far showed limited efficacy or induced rapid development of resistance. We critically need multi-targeted drug combinations, which shall selectively inhibit the

cancer cells and block the emergence of drug resistance. Utilizing pharmacological screening data from cancer samples, we have developed a logic-based network modeling approach called TIMMA to predict effective drug combinations. The TIMMA algorithm starts by identifying a set of essential drug targets that are most predictive of monotherapy responses. A drug combination is then treated as a combination of the essential targets, the effect of which can be estimated based on the set relationships with the observed target profiles. The TIMMA approach has been applied on the MDA-MB-231 triple-negative breast cancer cell line using 41 drugs and 384 targets. The predicted drug synergy scores were found significantly correlated with the experimental validation results. To further facilitate the statistical testing of drug combination experiment data, we have also developed a novel mathematical model called ZIP to score the drug interactions. Compared to the existing models such as Loewe additive and Bliss independence models, the ZIP model captures the drug interaction relationships by comparing the change in the potency and shape of the dose-response curves between individual drugs and their combination. We utilized a Delta score to quantify the deviation from the expectation of zero interaction, and proved that a non-interaction is equivalent to both probabilistic independence and dose additivity. Using data from a large-scale anticancer drug combination experiment, we demonstrated how the ZIP model captures the experimentally confirmed drug synergy while keeping the number of false positive lower than with the other scoring models. Further, rather than relying on a single parameter to assess drug interaction, we proposed the use of an interaction landscape over the full dose-response matrix to identify and quantify synergistic and antagonistic dose regions. Taken together, the computational-experimental pipeline for drug combination discovery offers an increased power to predict and test the most potential drug combinations and finally translate into treatment options by clinical collaborators.

10 Applied mathematics IV, optimization with applications

Organized by: Marko Mäkelä (TY).

Kaisa Joki (TY): Proximal bundle method for nonsmooth DC optimization

Joint work with A.M. Bagirov (Federation University Australia), N. Karmitsa (TY) and M.M. Mäkelä (TY).

The new bundle method is designed to solve unconstrained DC minimization problems of the form

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \Big\{ f(\boldsymbol{x}) = f_1(\boldsymbol{x}) - f_2(\boldsymbol{x}) \Big\}, \tag{8}$$

where the objective $f : \mathbb{R}^n \to \mathbb{R}$ is a difference of two convex functions f_1 and f_2. Such a function f is called a *DC function* and functions f_1 and f_2 are *DC*

components of f. Note that the function f is typically nonconvex and it does not need to be differentiable. Due to this, we cannot use the classical theory of optimization or smooth gradient based methods. However, a nice feature of DC functions is that for them it is still possible to utilize convex analysis and optimization to some extent. For example, a point $\boldsymbol{x}^* \in \mathbb{R}^n$ is an *ε-critical point* of $f = f_1 - f_2$ if for $\varepsilon \geq 0$ it satisfies the condition [1]

$$\partial_\varepsilon f_1(\boldsymbol{x}^*) \cap \partial_\varepsilon f_2(\boldsymbol{x}^*) \neq \emptyset, \tag{9}$$

where the set [2]

$$\partial_\varepsilon f_i(\boldsymbol{x}) = \left\{ \boldsymbol{\xi}_\varepsilon \in \mathbb{R}^n \mid f_i(\boldsymbol{y}) \geq f_i(\boldsymbol{x}) + \boldsymbol{\xi}_\varepsilon^T(\boldsymbol{y} - \boldsymbol{x}) - \varepsilon \text{ for all } \boldsymbol{y} \in \mathbb{R}^n \right\}$$

for $i = 1, 2$, is the *ε-subdifferential* of a convex function f_i at $\boldsymbol{x} \in \mathbb{R}^n$. The condition (9) is now a necessary condition for local optimality in DC programming.

The class of DC functions is very broad. For example, any twice continuously differentiable function can be represented as a DC function. Moreover, any continuous function can be approximated by the sequence of DC functions [3]. In addition many optimization problems of potential interest can be expressed into the form of a DC program such as production-transportation planning, location planning, cluster analysis, multilevel programming and multi-objective programming.

In this paper, our aim is to design a version of the bundle method to locally solve the unconstrained DC programming problem (8). The bundle method and its variations are among the most efficient methods in nonsmooth optimization (see [4] and references therein). To date most of these methods are based on the convex model of a function, also in the nonconvex case. Unfortunately, in the nonconvex case, a convex model might fail to describe the actual behavior of the objective. To the best of our knowledge, versions of the bundle methods based on the explicitly known DC structure of a problem have not been studied in depth before, even though this way we can take into account both the convex and the concave behavior of f.

Therefore, unlike the other known bundle methods, our new method utilizes explicitly the DC decomposition of the objective f in the model construction. The main idea is to approximate the subdifferentials of both DC components with a bundle. This means that we are maintaining two separate bundles which consists of subgradients from some neighborhood of the current iteration point. This subgradient information is used to construct separately a classical convex cutting plane model (see, e.g., [5]) for each DC component and by combining these approximations we obtain a new piecewise linear cutting plane model of f. It is worth noting that this new model is nonconvex and due to this it can describe better the actual behavior of the nonconvex objective than a convex model.

Our algorithm for DC programming is designed based on this new approach and it is also inspired by the methods introduced in [6, 7, 8]. In addition, we have proved the global convergence of the method to an ε-critical point after a finite number of steps. The results of the numerical experiments also show the good performance of the new method. For example our method uses quite often the least computational effort when compared to five other methods. Another interesting

fact is that, although our new algorithm is only a local method, it finds nearly always the global solution in our test problems. Due to this, our new method has a better ability to find global minimizers than the other five methods tested.

References

[1] W. Y. Sun, R. J. B. Sampaio, and M. A. B. Candido. Proximal point algorithm for minimization of DC functions. *J. Comput. Math.*, 21(4):451–462, 2003.

[2] R. T. Rockafellar. *Convex Analysis.* Princeton University Press, Princeton, New Jersey 1970.

[3] H. Tuy. *Convex Analysis and Global Optimization.* Kluwer Academic Publishers, Dordrescht, 1st edition, 1998.

[4] A. M. Bagirov, N. Karmitsa, M. M. Mäkelä. *Introduction to Nonsmooth Optimization: Theory, Practice and Software.* Springer, Cham, Heidelberg, 2014.

[5] K. C. Kiwiel. Proximity control in bundle methods for convex nondifferentiable minimization. *Math. Prog.,* 46:105–122, 1990.

[6] A. Fuduli, M. Gaudioso, and G. Giallombardo. A DC piecewise affine model and a bundling technique in nonconvex nonsmooth minimization. *Optim. Methods Softw.*, 19(1):89–102, 2004.

[7] A. Fuduli, M. Gaudioso, and G. Giallombardo. Minimizing nonconvex nonsmooth functions via cutting planes and proximity control. *SIAM J. Optim.*, 14(3):743–756, 2004.

[8] A. Fuduli, M. Gaudioso, and E. A. Nurminski. A splitting bundle approach for non-smooth non-convex minimization. *Optimization*, 2013.
DOI 10.1080/02331934.2013.840625

Jussi Hakanen (JY): Interactive multiobjective optimization: solution approaches and applications

Background

Typically, real-world optimization problems contain multiple conflicting objectives that need to be optimized simultaneously. These multiobjective optimization (MO) problems have several Pareto optimal (PO) solutions where none of the objectives can be improved without impairing some other objective. Mathematically all Pareto optimal solutions are equally good for the MO problem and further ordering them requires some additional information. Solving MO problems can mean different thing for different people: 1) identifying all PO solutions, 2) finding a representation for the Pareto optimal front, i.e., PO solutions in the objective space or 3) finding a most preferred solution for a human decision maker (DM) who can express preference information related to Pareto optimal solutions. During the last 50 years, many different types of methods have been developed for solving

MO problems both in the fields of multiple criteria decision making (MCDM) and evolutionary multiobjective optimization (EMO) (see e.g. [1, 2, 4]). Typically, MCDM methods combine DM preferences in order to find a most preferred PO solution while the goal for population based EMO methods have been to find a representation of the whole Pareto front. MCDM methods typically work with one solution at a time, assume certain properties from the considered functions and are guaranteed to produce PO solutions. On the other hand, EMO methods are population based, don't assume anything of the functions and don't have any guarantee on Pareto optimality of the solutions produced.

Interactive methods

Recently, EMO has also brought DM preferences into its solution process [5] and there has been lots of approaches to combine the benefits of both MCDM and EMO methods. There are different ways of considering DM preferences in the solution process: 1) before any optimization has been done, 2) a posteriori after a (large) set of PO solutions have been computed or 3) interactively where the phases of optimization and preference elicitation alternate [4]. In this paper, our aim is to find a most preferred PO solution by using interactive methods that have been found promising for solving real-world problems. In interactive methods, the solution process is iterative. First some PO solution(s) is computed and it is shown to the DM who evaluates it. If it is not preferred to her/him, then (s)he is asked to indicate how it should be improved. Then, this information is taken into account when new PO solution(s) are computed. The new solution(s) are then shown to the DM and the solution process continues until the DM has found the most preferred solution. The benefits of interactive methods include that PO solutions are computed only in the regions of the Pareto front that the DM is interested in. In addition, through the iterative process of expressing preferences and seeing the corresponding PO solutions computed, the DM can learn about the trade-offs between conflicting objectives and about her/his own preferences. Originally, interactive multiobjective optimization methods have been developed in MCDM but, recently, interactive EMO methods have also been developed.

Some applications

We illustrate the solution process of interactive methods with some applications related to dynamic processes. The first case study is related to wastewater treatment plant operation [3] where the main trade-off exists between the quality of the treated water and the operating costs resulting into a five objective problem. The second case study is related to optimizing dynamic chemical processes that include delayed effects which requires both pre and post processing of the process data in optimization [6]. In addition, interactive methods can be used to validate the optimization problem formulation and detect bad formulations that might not be noticed by using non interactive approaches. In both cases, the DM was able to conveniently study the inherent trade-offs and identify the most preferred PO solution through the interactive solution process.

References

[1] J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors. *Multiobjective Optimization: Interactive and Evolutionary Approaches*. Springer-Verlag, Berlin, 2008.

[2] K. Deb. *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Ltd., Chichester, 2001.

[3] J. Hakanen, K. Sahlstedt, and K. Miettinen. Wastewater treatment plant design and operation under multiple conflicting objective functions. *Environmental Modelling & Software*, 46:240–249, 2013.

[4] K. Miettinen. *Nonlinear Multiobjective Optimization*. Kluwer Academic Publishers, Boston, 1999.

[5] R. Purshouse, K. Deb, M. Mansor, S. Mostaghim, and R. Wang. A review of hybrid evolutionary multiple criteria decision making methods. In *2014 IEEE Congress on Evolutionary Computation (CEC)*, pages 1147–1154. IEEE, 2014.

[6] K. Sindhya, V. Ojalehto, J. Savolainen, H. Niemistö, J. Hakanen, and K. Miettinen. Coupling dynamic simulation and interactive multiobjective optimization for complex problems: An APROS-NIMBUS case study. *Expert Systems with Applications*, 41:2546–2558, 2014.

Timo Ranta (TTY): Elinkeinoelämän kanssa yhteistyössä toteutettuja optimointiprojekteja

Esitykseni käsittelee Tampereen teknillisen yliopiston Porin laitoksella toteutettuja optimointiprojekteja. Optimointiprojekteissa on tehty läheistä yhteistyötä alueen elinkeinoelämän – teollisuuden, yritysten ja julkisten toimijoiden – kanssa.

Esittelen TTY Porin laitoksella vuosien 2014–2015 aikana toteutettua hanketta – Optimointi ja paikkatieto yritysten toiminnan kehittämisessä ja tehostamisessa. Hankkeessa edistettiin optimoinnin ja paikkatiedon hyödyntämistä Satakunnassa. Pääkohderyhmä oli PK-yritykset. Hankkeessa kerättiin optimoinnista ja paikkatiedosta kiinnostuneet tekijät ja toimijat yhteen sekä innovoitiin asiaan liittyviä tutkimus-, liiketoiminta- ja kehittämismahdollisuuksia. Satakunnan alueella toimivien yritysten vaatimusten perusteella suunnitelluissa ja toteutetuissa piloteissa kehitettiin ja tehostettiin yritysten toimintaan liittyviä prosesseja, toimintoja ja infrastruktuuria. Toteutetut optimointi- ja paikkatietopilotit toimivat esimerkkinä yliopistoyhteistyön mahdollisuuksista alueen yrityksille.

Esittelen myös toista TTY Porin laitoksella tehtyä tutkimusta, missä optimointia on sovellettu käytetyn ydinpolttoaineen loppusijoituksen suunnitteluun.

Posiva Oy aikoo loppusijoittaa omistajiensa käytetyn ydinpolttoaineen kapseleissa Olkiluodon peruskallioon. Ennen loppusijoittamista käytettyjä polttoainenippuja säilytetään välivaraston vesialtaissa useita kymmeniä vuosia, jotta niiden

radioaktiivisuus ja jälkilämpöteho laskevat loppusijoituksen edellyttämälle tasolle. Halutut polttoaineniput valitaan välivarastosta ja kuljetetaan kapselointilaitokseen kapseleihin pakkaamista varten. Valmiit kapselit asennetaan loppusijoitustunnelien lattiaan porattuihin kapselireikiin.

Tutkimuksessa optimointia on sovellettu kahteen loppusijoitukseen liittyvään optimointitehtävään. Ensimmäisessä tehtävässä tavoitteena on minimoida loppusijoituksen kustannukset valitsemalla loppusijoitukselle aikataulu ja volyymi. Toisessa optimointitehtävässä loppusijoituksen aikataulu ja volyymi on kiinnitetty ja tavoitteena on valita polttoaineniput kuhunkin kapseliin siten, että suurin kapselikohtainen lämpöteho minimoituu.

Tutkimuksessa kehitetyllä mallilla on mahdollista tutkia, miten eri tekijät vaikuttavat loppusijoituksen kustannuksiin ja mikä ratkaisu on edullisin missäkin tilanteessa. Tulokset muun muassa osoittavat, että loppusijoituksen aikataululla ja volyymillä on merkittävä vaikutus kustannuksiin.

Jatkotutkimus keskittyy monitavoiteoptimoinnin soveltamiseen. Tutkimusta tehdään yhteistyössä Turun yliopiston optimointiryhmän kanssa.

11 Discrete mathematics I, words and tilings

Organized by: Ion Petre (ÅA), Tomi Kärki (TY).

Jarkko Peltomäki (TY): Sturmin sanat ja ketjumurtoluvut

Sturmin sanat ovat keskeinen tutkimuskohde sanojen kombinatoriikassa. Tavallisesti nämä sanat määritellään sellaisiksi (oikealta) äärettömiksi sanoiksi yli aakkoston $\{0,1\}$, joissa esiintyy täsmälleen $n+1$ pituutta n olevaa osasanaa eli tekijää. Sturmin sanojen syvällisten ominaisuuksien ymmärtämiseksi on kuitenkin usein välttämätöntä käsittää Sturmin sanat rotaatiosanoiksi.

Olkoon $\alpha \in (0,1)$ irrationaaliluku. Luvun $x \in [0,1)$ rata irrationaalisessa ympyrärotaatiossa $R(x) = \{x+\alpha\}$ koodataan seuraavalla tavalla binäärisanaksi $s_{x,\alpha}$:

$$s_{x,\alpha}(n) = \begin{cases} 0, \text{ jos } R^n(\{x+n\alpha\}) \in I_0, \\ 1, \text{ jos } R^n(\{x+n\alpha\}) \in I_1, \end{cases}$$

missä $I_0 = [0, 1-\alpha)$ ja $I_1 = [1-\alpha, 1)$ tai $I_0 = (0, 1-\alpha]$ ja $I_1 = (1-\alpha, 1]$. Jokainen Sturmin sana saadaan sanana $s_{x,\alpha}$ joillain (yksikäsitteisillä) luvuilla x ja α. Luku α vastaa kirjaimen 1 tiheyttä Sturmin sanassa $s_{x,\alpha}$.

On osoittautunut, että luvun α ketjumurtolukukehitelmä on erittäin hyödyllinen Sturmin sanaa $s_{x,\alpha}$ tutkittaessa. Käyn esitelmässäni läpi Sturmin sanojen perusominaisuuksia sekä muutamia ketjumurtolukujen sovelluksia Sturmin sanoihin. Hahmottelen millä tavoin ketjumurtolukuja käyttäen voi luonnehtia Sturmin sanoissa esiintyvät toistot eli tyyppiä u^2, u^3, ... olevat tekijät. Näistä pohdinnoista seuraa suoraan ketjutermien avulla ilmaistava kaava Sturmin sanan kriittiselle eksponentille. Edellä mainitun lisäksi kuvailen miten ketjumurtolukujen avulla

saadaan tietoa Sturmin sanojen abelin toistoista (abelin toisto on kuten tavallinen toisto $u_1u_2 \dots u_n$, mutta sanojen u_i kirjaimia voidaan permutoida mielivaltaisella tavalla).

Aleksi Saarela (TY): k-abelian equivalence and the theorem of Morse and Hedlund

The classical theorem of Morse and Hedlund states that an infinite word is ultimately periodic if and only if it has, for some n, at most n different factors of length n. Words that have exactly $n+1$ factors of length n for every n are called Sturmian words.

We consider generalizations of these facts in the case of k-abelian equivalence. For a positive integer k, two finite words u and v are defined to be k-abelian equivalent if every factor of length at most k appears as many times in u as in v. These equivalence relations form an infinite hierarchy of congruences bridging the gap between abelian equivalence ($k = 1$) and equality ($k \to \infty$).

The factor complexity of an infinite word w is a function that maps a positive integer n to the number of length-n factors of w. The k-abelian complexity $\mathcal{P}_w^k$ is defined in the same way, except that factors are counted modulo k-abelian equivalence. Now a k-abelian version of the theorem of Morse and Hedlund can be stated as follows: An infinite word w is ultimately periodic if $\mathcal{P}_w^k(n) < \min\{n+1, 2k\}$ for some n. Furthermore, if w is aperiodic, then it is Sturmian if and only if $\mathcal{P}_w^k(n) = \min\{n+1, 2k\}$ for all n.

We consider several results about the possible behaviors of k-abelian complexity functions. One example is that, unlike in the case of the usual factor complexity, there is no "complexity gap" above bounded complexity functions, because there are words with unbounded but arbitrarily slowly growing k-abelian complexities.

Michal Szabados (TY): An algebraic geometric approach to Nivat's conjecture

Joint work with Jarkko Kari.

We study colorings of the d-dimensional lattice $\mathbb{Z}^d$ by finitely many colors, these objects are also known as *multidimensional words* or *configurations*. A configuration is *periodic* if there exists a non-zero vector such that any two positions which differ by the vector have the same color. Denote by $P(n_1, \dots, n_d)$ the number of distinct rectangular patterns of size $n_1 \times \cdots \times n_d$ appearing inside the configuration. This P is called the *complexity function*. In one dimension, the famous Morse-Hedlund theorem relates complexity and periodicity: a sequence of colors is non-periodic iff $\forall n\colon P(n) \geq n+1$. We are interested in the two-dimensional generalization known as Nivat's conjecture: if a two-dimensional configuration is non-periodic, then $\forall m, n\colon P(m, n) \geq mn+1$. In our work we represent configurations as formal power series over d variables and study their annihilator ideal – polynomials, for which the product with the power series is zero. We prove that if $P(n_1, \dots, n_d) \leq n_1 \cdots n_d$, then the configuration can be written as a sum of periodic configurations. We use

Hilbert's Nullstellensatz in the proof. In two dimensions, we prove asymptotic version of Nivat's conjecture: if a two-dimensional configuration is non-periodic, then $P(m,n) \geq mn+1$ holds for all up to finitely many pairs m, n. These results have been presented at ICALP 2015 conference.

12 Discrete mathematics II, coding theory

Organized by: Tero Laihonen (TY).

Janne I. Kokkala (AYO): Binary codes and colorings of cube-like graphs

A cube-like graph is a Cayley graph for $\mathbb{Z}_2^n$. In particular, the kth power of the n-cube, Q_n^k, is the graph over $\mathbb{Z}_2^n$ that has edges between two vertices if their Hamming distance is at most k. An independent set in Q_n^k corresponds to a binary code of length n and minimum distance $k+1$. The chromatic number $\chi(Q_n^k)$ is thus the number of such codes needed to partition the n-dimensional Hamming space.

The problem of determining the chromatic number of Q_n^2 has attracted interest in coding theory and combinatorics. One application is channel assignment in hypercube networks [1]. It is known that asymptotically $\chi(Q_n^2) \sim n$ [2], but for small n, exact values of $\chi(Q_n^2)$ have been known only for $n \leq 7$. The maximum size of a one-error-correcting binary code of length 8 is 20, which gives a lower bound $\chi(Q_8^2) \geq \lceil 2^8/20 \rceil = 13$. On the other hand, 14-colorings were found independently by Hougardy in 1991 [3] and Royle in 1993 [4, Section 9.7]. We discuss a computer search that was used to find a 13-coloring of Q_8^2, thus proving that $\chi(Q_8^2) = 13$.

This is joint work with Patric R. J. Östergård.

References

[1] P.-J. Wan. Near-optimal conflict-free channel set assignments for an optical cluster-based hypercube network. *Journal of Combinatorial Optimization*, 1:179–186, 1997.

[2] P. R. J. Östergård. On a hypercube coloring problem. *Journal of Combinatorial Theory, Series A*, 108:199–204, 2004.

[3] G. M. Ziegler. Coloring Hamming graphs, optimal binary codes, and the 0/1-Borsuk problem in low dimensions. In H. Alt, editor, *Computational Discrete Mathematics*, volume 2122 of *Lecture Notes in Computer Science*, pages 159–171. Springer Berlin Heidelberg, 2001.

[4] T. R. Jensen and B. Toft. *Graph Coloring Problems*. Wiley, New York, 1995.

Ville Junnila (TY):
On information retrieval in associative memories

In current memory systems, there are two main problems. The first problem is how large amount of data can be stored and the second one is how information can be efficiently retrieved from the memory systems. The recent technological development has given satisfactory solutions for the first problem. One way to organize information for efficient retrieval is the use of associative memories. Associative memories provide to the problem an approach mimicking human-like memories, where information units are linked to each other if they are associated and information is retrieved from memories based on these associations. First attempts into this direction have been presented in a paper by Yaakobi and Bruck (2012).

The problem of information retrieval in associative memories can be modeled by a graph $G = (V, E)$ as follows. The vertices of the graph represent the memory entries, in which the information units are stored, and the edges between vertices represent the associations of information units to each other. Then a subset of vertices, i.e., a code in V, is chosen as a set of attributes, which is also called a reference set, according to which the information is retrieved. More precisely, m hints from the set of attributes are given and then the retrieved set of information units is formed by the vertices associated to all the hints. The maximum size of a retrieved set is called the *uncertainty* of the code in the graph (or memory) G. In this presentation, we focus on the case where there is no uncertainty, i.e., the information unit can be retrieved unambiguously.

One natural way to measure the quality of the code is the maximum number m_u of hints needed to uniquely determine the sought information unit. Hence, we are interested in constructing codes with as small m_u as possible. In the presentation, this is more closely considered in the binary Hamming spaces. Moreover, we also consider the connection of information retrieval in associative memories to other applications such as Levenshtein's sequences reconstruction problem and sensor network monitoring.

Roope Vehkalahti (TY):
Capacity and geometry of numbers in fading channels

During the last fifteen years multiple-input multiple-output (MIMO) channels have slowly replaced single antenna channels as a main subject of study in information theory. In such channel the message signal is transmitted from multiple antennas unlike in the traditional one-antenna transmission. In addition the system may also contain several receiving antennas. Interest on such channels faced a sudden rise of interest when in 1999 Telatar proved [2] that in the presence of Gaussian noise and ergodic fading the capacity of multiple antenna channel is considerably higher than that of a single antenna system. Currently most of the wireless standards support MIMO transmission.

One of the key challenges in all of information theory is to build capacity achieving structured codes. So far, and in most MIMO channel models, known

coding strategies leave at least a constant gap to capacity.

In the case of classical single antenna Gaussian channels there exist a rich theory of lattice codes developed to attack these questions. In the heart of this theory are sphere packing arguments that prove that the performance of a lattice code can be roughly estimated by the size of a geometrical invariant of the lattice. This connection has been extremely fruitful and has motivated a monumental work connecting algebra, geometry and information theory.

In [1] we prove that such theory does exists also in fading MIMO channels and that it forms almost an exact analogue to the classical Gaussian case. Based on this observation, we develop a general theory of lattices for fading channels and connect capacity questions and geometric properties of multiple antenna lattices codes.

Building on this general theory and on number theoretic existence results we construct and analyze capacity approaching coding schemes for several single user fading channel models.

The talk is based on joint work [1] with Laura Luzzi.

References

[1] L.Luzzi and R. Vehkalahti, "Almost universal codes achieving ergodic MIMO capacity within a constant gap", submitted, draft at http://arxiv.org/abs/1507.07395.

[2] E. Telatar, "Capacity of multi-antenna Gaussian channels", *Europ. Trans. Telecomm.*, vol. 10, no. 6, pp. 585–595, Nov.-Dec. 1999.

13 Discrete mathematics III, algebraic combinatorics and its applications

Organized by: Ragnar Freij-Hollanti, Thomas Westerbäck (AYO).

Juho Lauri (TTY): Rainbow colorings

A path in an edge-colored graph is said to be rainbow if no two edges of it have the same color. A graph is rainbow connected if there is a rainbow path between every pair of its vertices. Besides being an interesting way of strengthening the connectivity property, rainbow connectivity also finds applications in the transfer of sensitive information, message routing, and layered encryption.

The rainbow connection number of a graph G, denoted by rc(G), is the minimum number of colors needed to rainbow connect G. Determining rc(G) for a given graph G is computationally hard, even for very restricted graph classes. In this talk, we give an introduction to rainbow coloring. In particular, we focus on the rainbow connection number along with its most prominent variants which consider

vertex-colorings and/or so-called strong colorings. We conclude with open problems, some with a computational flavor.

Emanuele Ventura (AYO): The poset of proper divisibility

We study the partially ordered set of all multidegrees of monomials which strictly divides a given monomial. We prove that its order complex is (non-pure) shellable, using the theory of CL-shellability. Along the way, we exhibit an example of a poset which is dual CL-shellable that is not CL-shellable itself. For special cases, we provide the ranks of all homology groups and give a succinct formula for the Euler characteristic of these simplicial complexes. This is a joint work with A. Macchia, D. Bolognini and V. Welker.

Kaie Kubjas (AYO): The geometry of positive semidefinite rank

Positive semidefinite rank is a generalization of regular and nonnegative matrix ranks. We are interested in the semialgebraic set and boundaries of the set of matrices of rank at most r and of psd rank at most k, where r and k are small. In particular, we describe the algebraic boundary of this semialgebraic set for $r = 3$ and $k = 2$ and conjecture a characterization of the boundary for $r = k + 1$. I will also explain a geometric version of our conjecture in terms of polytopes and spectrahedral shadows. This talk is based on joint work with Elina Robeva and Richard Z. Robinson.

14 Discrete mathematics IV, algebra and applications

Organized by: David Karpuk, Marcus Greferath, Oliver W. Gnilke, Ha Tran (AYO).

Kimmo Järvinen (AYO): Towards practical homomorphic encryption with hardware acceleration

Fully homomorphic encryption that allows arbitrary computations with encrypted data offers great promises for secure and privacy-aware cloud computing. Despite major advantages of the recent years, contemporary homomorphic encryption schemes still have enormous computation and communication requirements. In this talk, we survey some recent advantages and discuss improving the feasibility of homomorphic encryption with dedicated hardware accelerators. The talk is partly based on the work presented at CHES 2015.

Erkko Lehtonen (Technische Universität Dresden): Reconstructing functions from identification minors

We consider functions of several arguments from A to B, i.e., mappings $f\colon A^n \to B$ for some positive integer n, called the *arity* of f. Let us denote by $\binom{n}{2}$ the set of all 2-element subsets of $\{1,\dots,n\}$. For any $I = \{i,j\} \in \binom{n}{2}$ with $i < j$, let $f_I\colon A^{n-1} \to B$ be the function given by the rule

$$f_I(a_1,\dots,a_{n-1}) = f(a_1,\dots,a_{j-1},a_i,a_j,\dots,a_{n-1}),$$

for all $a_1,\dots,a_{n-1} \in A$. (Note that a_i occurs twice on the right side of the above equality: both at the i-th and the j-th position.) Such a function is referred to as an *identification minor* of f. We say that n-ary functions f and g are *equivalent,* and we write $f \equiv g$, if there exists a permutation σ of $\{1,\dots,n\}$ such that $f(a_1,\dots,a_n) = g(a_{\sigma(1)},\dots,a_{\sigma(n)})$ for all $a_1,\dots,a_n \in A$.

These notions give rise to the following reconstruction problem: Is a function $f\colon A^n \to B$ uniquely determined, up to permutation of arguments, by the collection of its identification minors? In order to discuss whether and to which extent functions are reconstructible from their identification minors, let us recall some standard terminology of reconstruction problems in our setting. Let $f\colon A^n \to B$. The *deck* of f is the multiset $\langle f_I/{\equiv} \mid I \in \binom{n}{2}\rangle$ of the equivalence classes of the identification minors of f, and its elements are referred to as the *cards* of f. A function $g\colon A^n \to B$ is a *reconstruction* of f if the decks of f and g are identical. If f is equivalent to all its reconstructions, we say that f is *reconstructible.* Let $\mathcal{C} \subseteq \bigcup_{n\geq 1} B^{A^n}$. We say that $\mathcal{C}$ is *reconstructible* if all its members are reconstructible. We say that $\mathcal{C}$ is *weakly reconstructible* if for every $f \in \mathcal{C}$, all reconstructions of f that are members of $\mathcal{C}$ are equivalent to f.

We present some results – both positive and negative – and open problems concerning the reconstruction problem of functions and identification minors. The following classes of functions are known to be reconstructible: totally symmetric functions [6], affine functions over finite fields [7]. The following classes of functions are known to be weakly reconstructible: affine functions over a cancellative nonassociative right semiring [7], linear functions over a nonassociative right semiring [7], functions determined by the order of first occurrence (submitted). (In the above discussion, functions are assumed to have a sufficiently large arity.) Infinite families of non-reconstructible functions were described by Couceiro, Schölzel, and the current author in [3]. (This work had a surprising connection to the reconstruction problem of hypergraphs and one-vertex-deleted subhypergraphs, and we provided infinite families of non-reconstructible hypergraphs that were significantly different from those presented by Kocay [4] and Kocay and Lui [5].) In [2], we also considered the related set-reconstruction problem, where the deck is defined as a set (as opposed to a multiset) of cards.

A related problem is to determine which functions $f\colon A^n \to B$ have a *unique identification minor,* i.e., satisfy the condition that $f_I \equiv f_J$ for all $I, J \in \binom{n}{2}$. This problem was previously posed by Bouaziz, Couceiro, and Pouzet [1]. Examples of functions with a unique identification minor include the 2-set-transitive functions (implicit in [1]; for a proof, see, e.g., [6]), functions determined by the order of

first occurrence [8], and functions determined by content and singletons (work in progress). Whether there exist further examples of such functions remains an open problem.

References

[1] M. Bouaziz, M. Couceiro, M. Pouzet, Join-irreducible Boolean functions, *Order* **27** (2010) 261–282.

[2] M. Couceiro, E. Lehtonen, K. Schölzel, Set-reconstructibility of Post classes, *Discrete Appl. Math.* **187** (2015) 12–18.

[3] M. Couceiro, E. Lehtonen, K. Schölzel, Hypomorphic Sperner systems and non-reconstructible functions, *Order* **32** (2015) 255–292.

[4] W. L. Kocay, A family of nonreconstructible hypergraphs, *J. Combin. Theory Ser. B* **42** (1987) 46–63.

[5] W. L. Kocay, Z. M. Lui, More non-reconstructible hypergraphs, *Discrete Math.* **72** (1988) 213–224.

[6] E. Lehtonen, Totally symmetric functions are reconstructible from identification minors, *Electron. J. Combin.* **21**(2) (2014) #P2.6.

[7] E. Lehtonen, Reconstructing multisets over commutative groupoids and affine functions over nonassociative semirings, *Internat. J. Algebra Comput.* **24** (2014) 11–31.

[8] E. Lehtonen, On functions with a unique identification minor, *Order* (2015), DOI:10.1007/s11083-015-9352-1.

Alex Karrila (AYO): Hadamard rotations and reliability in wireless communications

In wireless communications, messages are blurred by the random fading and noise of the electric field, and the transmission power of the devices is restricted. This makes the reliability of communications an important issue in code design. For certain reliability and decoding efficiency reasons, wireless communications conventionally relies on *cubic lattice codes*. Roughly speaking, this means that n subsequent values of the transmitted electric field are interpreted as the coordinates of a vector in a bounded sending region carved out of a rotated cubic lattice $\mathbb{Z}^n$, and each lattice vector in the sending region is labeled with a bit sequence. The objective of *physical-layer reliability design* is then to find the rotation that maximizes the probability that the receiver decodes the transmitted lattice point correctly, having a received a distorted version.

In this talk, we study physical-layer reliability as a geometric lattice design question. We mainly consider *fast fading single-input single-output (SISO) channels*

with an (almost) arbitrary distribution of the random fading. We discuss heuristics, rigorous probability bounds, and numerical simulation results suggesting the suitability of *Hadamard rotations* for *non-outage fading models* such as *the Rician fading*. A rotation matrix $W \in \mathbb{R}^{n\times n}$ is Hadamard if it satisfies the additional condition that $\sqrt{n}W_{ij} = \pm 1$ for all $1 \leq i, j \leq n$. Time permitting, we may consider generalizations to more complicated channel models depicting an environment with quasi-static electric-field scatterers and/or multiple antennas, or applications of Hadamard matrices in physical-layer security. The talk concerns physical-layer design, and should not be confused with the well-known *Walsh-Hadamard error correction codes*.

This talk is based on an ongoing work with Oliver W. Gnilke, Camilla Hollanti, David Karpuk, Padraig Ó Catháin, and Niko Väisänen.

15 Education I, matematiikan oppiminen yliopistossa

Organized by: Juha Oikkonen, Johanna Rämö, Terhi Hautala (HY).

Jokke Häsä, Sini Karppinen, Juulia Lahdenperä, Lotta Oinonen (HY): Vallankumous matematiikkabasaarissa – opiskelijat ohjaajina ja vertaispalautteen antajina

Matematiikkabasaari on avoin ja opiskelijakeskeinen oppimisympäristö Helsingin yliopiston matematiikan ja tilastotieteen laitoksella. Matematiikkabasaarissa opiskellaan matematiikkaa sekä yhdessä että yksin, puhumalla, lukemalla, piirtämällä ja kirjoittamalla, aamusta iltaan. Basaari levittäytyy laitoksen pääkäytävälle, jonka seiniä peittävät isot liitutaulut ja jonka valkotaulupintaisiin pöytiin voi piirtää. Puheensorinan keskellä kiertelee värikkäisiin huomioliiveihin pukeutuneita ohjaajia.

Opiskelijat luovat matematiikkabasaarin joka päivä uudelleen paitsi kokoontumalla sinne opiskelemaan, myös toimimalla basaarissa ohjaajina. Ohjaajien työhön kuuluu ohjaamisen lisäksi viikoittaisia palavereja, joissa pohditaan matemaattisten kysymysten ohella ohjaamiseen liittyviä asioita. Ohjaajien kanssa käytyjen keskustelujen tuloksia ovat esimerkiksi seuraavat opiskelijakeskeisen ohjaamisen periaatteet: (1) Kuuntele. (2) Ohjaa yksilöllisesti. (3) Anna opiskelijan tehdä ja oivaltaa itse. (4) Kannusta. (5) Ole aktiivinen. (6) Jaa huomiosi. (7) Auta lukemaan kurssimateriaalia. (8) Ohjaajan ei tarvitse tietää kaikkea. (9) Opeta opiskelutaitoja. (10) Älä ota tunteenpurkauksia henkilökohtaisesti. (11) Kannusta yhteistyöhön.

Ohjaajaksi haluavia opiskelijoita on joka lukukausi ollut enemmän kuin olemme voineet työllistää. Järjestimme syksyllä 2015 kurssin Opiskelijalähtöinen ohjaaminen, jotta mahdollisimman monet opiskelijamme voisivat saada kokemusta ohjaamisesta. Tällä kurssilla opiskelijat pääsivät harjoittelemaan ohjaamista Matem-

atiikka tutuksi -kurssin pienryhmätapaamisissa, joita he ohjasivat muutaman hengen tiimeinä. Kurssiin kuului myös viikoittaisia tapaamisia, joissa opiskelijat tutustuivat opiskelijalähtöiseen ohjaamisen periaatteisiin erilaisista näkökulmista ja pohtivat omaa toimintaansa. Kurssin kokonaisvaltaisena tavoitteena oli antaa opiskelijoille mahdollisuus osallistua aktiivisesti laitoksemme opetuksen järjestämiseen heti opintojen ensimmäisistä vuosista lähtien.

Opiskelijalähtöistä oppimista voidaan toteuttaa myös vertaispalautteen kautta. Siinä opiskelijat arvioivat ja antavat palautetta toistensa työstä opettajan laatimien ohjeiden ja kysymysten avulla. Tutkimusten mukaan vertaispalautteen antaminen aktivoi opiskelijan arvioimaan kriittisesti myös omaa työtään. Sen vuoksi vertaispalauteharjoitukset opettavat paitsi palautteen antamista ja vastaanottamista, myös itsearviointitaitoja ja sitä kautta tukevat sekä välitöntä oppimista että yleisemmin opiskelutaitojen kehittymistä. Vertaispalautetta on sovellettu syksyllä 2015 kurssilla Tieteellinen viestintä opiskelijoiden kirjallisten töiden teossa. Kurssin tarkoituksena on kehittää opiskelijoiden viestinnän ja matemaattisen tekstin tuottamisen taitoja.

Esitelmässä kerromme, miten matematiikkabasaari, ohjaajien koulutus, opiskelijalähtöisen ohjaamisen kurssi ja vertaisarviointi on käytännössä toteutettu. Kerromme myös, kuinka kokeilut ovat onnistuneet, sekä pohdimme, mitkä asiat vielä vaativat kehittämistä.

Pekka Pankka (JY):
Kaksi opetuskokeilua erikoiskursseilla
Two teaching experiments on topics courses

Opetin syksyllä 2014 algebrallisen topologian perusteita käyttäen käänteisenluokkahuoneen menetelmää ja syksyllä 2015 algerallisen topologian erikoiskurssin hyödyntäen luentopäiväkirjaan perustuvaa suoritustapaa. Kerron kurssien käytännön totetuksesta, päämääristä ja tekemistäni havainnoista.

Fall 2014 I gave an introductory course on algebraic topology using flipped classroom method. Fall 2015 I used learning diaries on an advanced course in algebraic topology. I will discuss arrangements, goals, and observations related to these courses.

Riikka Kangaslampi, Harri Varpanen (AYO):
Opetuskokeiluja matematiikan massakursseilla
Teaching experiments on large mathematics courses

Kerromme kahteen Aalto-yliopiston kandiopiskelijoiden matematiikan kurssiin liittyvistä kokeiluistamme.

Kurssiin Lineaarialgebra ja differentiaaliyhtälöt liittyen kerromme kahdesta kokeilusta. Syksyn 2014 kurssilla opiskelijoilla oli mahdollisuus pitää oppimispäiväkirjaa. Tarkoituksena oli sekä kannustaa opiskelijoita, erityisesti niitä, joille kurssi on vaativa, syventymään aihepiireihin ja pohtimaan käsiteltävien asioiden

yhteyksiä ja merkitystä, että erityisesti tarjota luennoitsijalle ajantasaista tietoa siitä, miten opiskelijat ymmärtävät asioita. Kerromme tämän kokeilun saamasta palautteesta, hyödyistä ja haasteista. Syksyn 2015 kurssilla perinteinen välikoe korvattiin verkkokokeella. Kerromme hiukan sen toteutuksesta, mutta erityisesti siitä, miten verkkokokeen erilaisten tehtävien tulokset korreloivat perinteisen loppukokeen tulosten kanssa, ja millaista palautetta verkkokokeesta saatiin opiskelijoilta.

Kurssin Differentiaali- ja integraalilaskenta 1 osalta pohdimme kolmen luennointikerran pohjalta sitä, miten luennoida sadoille opintonsa aloittaville diplomi-insinööriopiskelijoille tehokkaasti 18 tunnissa koko yhden muuttujan sisältö – mukaanlukien sarjat ja differentiaaliyhtälöt sekä numeriikka – kun salissa ei ole taulua.

We will tell about our experiences related to two bachelor level mathematics courses at Aalto University.

Regarding the course Linear algebra and differential equations in Aalto University we will tell about two teaching experiments. Fall 2014 the students had the possibility to keep a learning diary. The aim of the learning diaries was on one hand to give instant feedback for the lecturer, and on other hand to help the students deepen their understanding of the concepts and theorems presented on the course. We will tell about the feedback, benefits and drawbacks of this experiment. Fall 2015 the traditional midterm exam was replaced by on online exam. We will tell a little bit about its implementation, but especially how results of different types of problems in the online exam correlate with the results of the traditional final exam, and what kind of feedback the students gave for the exam.

Regarding the course Differential and integral calculus 1 we will discuss, based on three years' experience, how to lecture in 18 hours the whole one-variable calculus – including series, differential equations, and numerics – for hundreds of fresh engineering students, when there is no black- or whiteboard in the lecture hall.

16 Education II, opetusteknologioiden hyödyntäminen

Organized by: Antti Rasila (AYO), Mika Hirvensalo (TY).

Kirsti Hemmi (ÅA): Matematiikan osaamisen tavoitteet ja opetusteknologian rooli

Käsittelen alustuksessani matematiikan opetuksen ?ikuisia kysymyksiä? joiden pohjalta voisimme keskustella miten tänä päivänä näihin kysymyksiin otetaan kantaa, kun suunnitellaan ja organisoidaan erilaisia oppimisympäristöjä. Miten huomioida erilaiset oppijat? Mitkä matematiikan sisällöt ovat keskeisiä nykypäivän opetuksessa (Mitä yksittäiset oppilaat/yhteiskunta tarvitsevat tulevaisuudessa)?

Miten oppimateriaalia ja opetusteknologiaa voidaan optimaalisesti käyttää hyväksi opetuksessa ja oppimisessa? Otan myös esille joitakin erityispiirteitä uudesta opetussuunnitelmasta ja siitä miten ne mahdollisesti vaikuttavat tulevien matematiikan opiskelijoiden valmiuksiin opiskella yliopistomatematiikkaa.

Simo Ali-Löytty, Panu Parviainen, Seppo Pohjolainen (TTY): Opetusteknologiaa hyödyntävä oppimisympäristö MATLABin alkeiden opiskeluun

MATLAB on maailmanlaajuisesti hyvin yleinen insinöörien ja tiedemiesten käytössä oleva laskenta- ja mallinnusohjelmisto. MATLABiin on olemassa suuri määrä teknilliseen laskentaan painottuvia lisäsovelluksia, minkä vuoksi MATLAB on myös Tampereen teknillisessä yliopistossa (TTY) hyvin laajasti käytetty työkalu. MATLABin käyttöä opetetaan lähes kaikille TTY:n opiskelijoille ja se on myös olennainen teknillisen laskennan ohjelmisto muutamilla TTY:n laitoksilla. Nykyään TTY myös kustantaa kaikille opiskelijoilleen ilmaisen MATLAB-lisenssin.

STACK (System for Teaching and Assessment using a Computer algebra Kernel) on tietokoneavusteinen matematiikan arviointityökalu. STACK-tehtävät voidaan yhdistää Moodle-tentteihin siten että tehtävässä on esimerkiksi tyhjiä vastauslaatikoita, joihin opiskelija kirjoittaa vastauksensa. Moodle on oppimisympäristö, joka on käytössä TTY:llä. STACK-tehtävätyypin etuina ovat muun muassa sen kyky osata lukea ja ymmärtää erilaisia matemaattisia syötteitä, satunnaistaa tehtävän lähtöarvot sekä antaa automaattista palautetta opiskelijan vastauksen oikeellisuudesta. Tällöin opettajan ei henkilökohtaisesti tarvitse arvioida vastauksia.

TTY:llä pilotoitiin syksyllä 2015 uudistettua MATLABin alkeiden kokonaisuutta. MATLABin alkeet on pakollinen osa ensimmäisen vuoden opiskelijoiden matematiikan peruskurssia. Alkeiden opettamisessa hyödynnettiin opetusteknologiaa aiempaa enemmän. Tätä opetusteknologiaa lähdettiin kokeilemaan, koska pyrittiin laatimaan opiskelijoille itsenäisesti suoritettavissa oleva kokonaisuus. Aiemmin MATLABin alkeita on ainoastaan opetettu tietokoneluokissa, joissa opiskelijat tekivät tehtäviä kahden tai kolmen hengen ryhmissä, jolloin yksilötason oppimista oli vaikeaa huomata. Tavoitteena on myös pitkällä aikavälillä kohdentaa pienentyviä opetusresursseja niitä tarvitseville opiskelijoille ja tehdä opinnoista kaiken kaikkiaan joustavampia. Opetusteknologia koostuu STACK-tehtävistä, opetusvideoista, ohjepdf-tiedostoista sekä EXAM-tentistä. Tämä materiaali lisättiin erilliselle kaikille opiskelijoille avoimelle Moodle-sivulle, jossa oli lisäksi linkkejä muuhun MATLAB-oppimateriaaliin ja keskustelupalstoja. Näiden lisäksi järjestettiin myös kontaktiopetusta yhden viikon aikana, jolloin opiskelijat saivat tehdä assistentin ohjaamana STACK-tehtäviä. Laadittiin myös sähköinen tentti TTY:n EXAM-järjestelmään niitä opiskelijoita varten, jotka eivät aikamääreeseen mennessä saaneet tehtyä tehtäviä hyväksytysti. Tänä vuonna tenttiä testattiin vain osalla opiskelijoista kokemusten saamiseksi. Tulevina vuosina sähköinen tentti pyritään saamaan osaksi kaikkien opiskelijoiden MATLABin alkeiden suoritusta.

Pilotin tavoitteena oli paitsi luoda kaikentasoisille opiskelijoille sopivat itsetarkastavat tehtävät, myös kehittää muutakin MATLABin oppimateriaalia niin että opetettavat asiat jäisivät myöhempiäkin opintoja varten jollain tasolla mieleen. MATLABin alkeiden suorittaminen on osa pakollisia opintoja kaikille noin 800:lle ensimmäisen vuoden opiskelijalle, joten itsestään tarkastavat tehtävät olivat tässä tapauksessa tervetulleita esimerkiksi opetusresurssien vähäisen määrän vuoksi. Työmäärältään alkeet ovat laajuudeltaan noin yksi opintopiste, mutta kokonaisuutta ei ole mahdollista suorittaa erikseen. Alkeiden opetettava sisältö pyrittiin valitsemaan mahdollisimman pitkälti niin, että se tukisi paitsi laskentaohjelmiston käytön alkeiden oppimista mutta tarjoaisi myös muissakin opinnoissa hyödyllisiä työkaluja. Päätettiin alkeiden oppimateriaalin koostuvan viidestä eri MATLABin osa-alueesta, jotka olivat peruskomennot, kuvaajien piirtäminen ja käsittely, editoriikkunan ja skriptojen käyttö, MATLABin omien funktioiden käyttö sovitteissa sekä tiedoston lukeminen ja käsittely.

TTY:n EXAM-järjestelmän sähköinen tentti painottui myös yllä kuvatuille MATLABin osa-alueille. Ennen tenttiä opiskelijaa kehotettiin opiskelemaan näiden osioiden asiasisältöä. Tentti koostui tenttijän opiskelijanumeroa hyödyntävistä MATLABin p-tiedostoista, joiden kautta tenttijä sai ohjeet ja tenttikysymykset. Vastaukset kirjoitettiin m-tiedostoon, joka palautettiin tenttivastauksen liitteenä. Tenttivastauksena annettiin lisäksi varmistuksena tentin suorituksesta komentoikkunaan ilmestyvä tuloste. Sähköinen tentti kehitettiin alkeiden suorittamista varten vararatkaisuna vielä tälle syksylle mutta tenttimismahdollisuutta parannetaan jatkoa varten tästä kokeilusta saatujen kokemusten perusteella.

Haasteellista materiaalin luomisessa oli keksiä tehtävien sisältö sellaiseksi, että se testaisi nimenomaan MATLABin käyttötaitojen oppimista, koska STACK on kuitenkin pohjimmiltaan matematiikan tulkitsemiseen luotu työkalu. Opiskelijoilta kerättiin myös sekä kvantitatiivista että kvalitatiivista palautetta opetusmateriaalista. Useassa sanallisessa palautteessa luki, että tehtävät oli liian helppo kopioida MATLABiin paljoa enempää ymmärtämättä, mitä tapahtuu. Toisaalta tehtävät eivät myöskään olleet ainakaan kaikkien mielestä täysin triviaaleja. Kehityskohde voisi esimerkiksi olla tehtävänantojen muuttaminen sanallisiksi eli sellaiseen muotoon, mistä ei pystyisi suoraan kopioimaan valmiita komentoja. Aikaisempiin vuosiin verrattuna MATLABin alkeiden läpipääsyprosentti säilyi suunnilleen samana, eli ei ollut merkittävää notkahdusta suuntaan tai toiseen. Kokonaisuudessaan pilotin ensimmäinen vaihe oli onnistunut ja sen esille tuomat kehityskohteet otetaan huomioon ennen pilotin toista vaihetta, joka toteutetaan syksyllä 2016. Automaattisesti itsensä tarkastavat tehtävät ovat suuren opiskelijamäärän vuoksi hyvin ideaalisia. Lisäksi kun alkeet sijaitsevat Moodlessa, niiden toteutus voidaan pitää avoimena ympäri vuoden. Tämän vuoksi opiskelijat voivat missä vaiheessa tahansa palata kertaamaan MATLABin alkeita.

Riikka Nurmiainen (Metropolia AMK): Opetusteknologiaa hyödyntäen kohti parempaa oppimista – matematiikan opiskelua Metropolia AMK:n talotekniikan tutkinto-ohjelmassa

Metropolian talotekniikan tutkinto-ohjelmassa on vuodesta 2010 matematiikan kotitehtävät toteutettu Moodlen STACK-tehtävinä. STACK-tehtävissä opiskelijoille generoituu omat lähtöarvot tehtäviin ja siten oikeat vastaukset eivät yleensä ole samat. Opiskelijat syöttävät vastauksensa sille varattuun kenttään ja ohjelma tarkastaa vastauksen. Opiskelijat ovat ottaneet STACK-tehtävät erittäin positiivisesti vastaan. Opettajan kannalta katsottuna STACK-tehtäviin siirtyminen on ollut onnistunut valinta: opiskelijat laskevat kotitehtäviä huomattavasti aiempaa enemmän, Moodleen tallettuu automaattisesti tieto lasketuista tehtävistä ja lisäksi opiskelijat oppivat mm. syntaksia ja laskujärjestyksen ohjaamista sulkeilla (esim. sqrt(2), xˆ(1/3), x/(y+z)) sekä merkitsevien numeroiden pohdintaa.

STACK-kotitehtäviä on julkistettu yleensä viikottain ja niille on asetettu määräaika. Tällöin opiskelijat tekevät töitä matematiikan eteen tasaisesti. Syksyyn 2015 asti opiskelijan arvosana on muodostunut laskettujen STACK-tehtävien, muiden tehtävien sekä muutaman pikkutestin perusteella. Jonkin verran on ollut havaittavissa kopiointia, vaikka lähtöarvot STACK-tehtävissä eivät ole samat. Lisäksi osa opiskelijoista ei ole kirjoittanut STACK-tehtävien ratkaisuja vihkoonsa, vaan muistiinpanot ovat olleet lähinnä pelkkiä laskimesta saatuja lukuarvoja.

Jotta oppiminen olisi entistä parempaa, on syksystä 2015 alkaen muutettu matematiikan opetusjärjestelyjä. Luennoista on pääosin luovuttu ja opiskelumateriaali on tarjolla ns. Talotekniikan matikkapankissa. Pankista löytyy eri aihealueisiin liittyvä teoria ja esimerkit pdf-tiedostoina sekä varsin kattavasti jo videoinakin. On siis otettu käyttöön käänteinen luokkahuone (flipped classroom): tunnille tullaan laskemaan. Tehtävät ovat pääasiassa STACK-tehtäviä. Opettaja kiertää ja auttaa ja tekee samalla omia muistiinpanoja opiskelijoiden työskentelystä. Opettaja ohjaa myös opiskelijoiden ”vihkotyöskentelyä”. Opiskelijat tekevät kurssin aikana itsearviointia. Muutama pikkutestikin pidetään – lähinnä opiskelijan oman itsearvioinnin tueksi. Alussa muutama opiskelija vastusti sitä, että opettaja ei enää opeta. Parin viikon opiskelun jälkeen he kuitenkin ilmoittivat, että eivät halua palata entiseen tapaan.

Viikottain julkistettaville STACK-tehtäville asetetut määräajat ovat toisille liian tiukkoja ja toiset haluaisivat edetä rivakammin. Seuraavana kehitysaskeleena on lisätä eritasoisia STACK-tehtäviä ja löytää lisää keinoja omatahtisen oppimisen mahdollistamiseen – kuitenkin niin, että opintojakso suoritetaan määräajassa.

Antti Valmari (TTY): MathCheck relation chain checker

Some students have very weak simplification skills. For instance, they compute $\frac{x}{2} + \frac{y}{3} = \frac{x+y}{2+3}$, $\sqrt{x+1} = \sqrt{x} + \sqrt{1}$, or $\sin(x+\pi) = \sin x + \sin \pi$. MathCheck is intended to help them improve their skills.

Consider the typical way of doing exercises in universities. Assume that a student has to simplify $\sin(\frac{\pi}{2}+x)+\sin(\frac{\pi}{2}-x)$. At home in the evening, he writes

$$\sin\left(\frac{\pi}{2}+x\right)+\sin\left(\frac{\pi}{2}-x\right) \;=\; \sin\frac{\pi}{2}+\sin x+\sin\frac{\pi}{2}-\sin x \;=\; 2\sin\frac{\pi}{2} \;=\; 2$$

in his notebook. Next day in the exercise session he hears that his answer is wrong, and the correct solution is shown to him.

MathCheck is at `http://www.cs.tut.fi/%7eava/mathcheck.html` at the time of writing this. The student can give his solution to it as follows:

```
sin(pi/2 + x) + sin(pi/2 - x) = sin pi/2 + sin x +
sin pi/2 - sin x = 2 sin pi/2 = 2
```

MathCheck replies:

$\sin(\frac{\pi}{2}+x)+\sin(\frac{\pi}{2}-x) \;=\; \sin\frac{\pi}{2}+\sin x+\sin\frac{\pi}{2}-\sin x$

Relation does not hold when $x = 1$

left $=$ 1.080605

right $=$ 2.

That is, the student learns already at home that his answer is wrong. He can then use a book of formulas, consult a fellow student, or browse the Internet to find information on how $\sin(\frac{\pi}{2}+x)$ and $\sin(\frac{\pi}{2}-x)$ should be simplified. If he has a guess about an individual step such as perhaps $\sin(\frac{\pi}{2}-x)=\cos(-x)=-\cos x$, he can try it with MathCheck. It reveals that $\sin(\frac{\pi}{2}-x)=\cos(-x)$ but $\cos(-x)\neq-\cos x$. Thanks to the feedback provided by MathCheck, the student can improve his solution at home until it is correct.

If the student needs a lot of exercise on, say, rational functions, the teacher can give him a set of 20 problems of increasing difficulty. The problems *need not* (and cannot) be fed into MathCheck. The student must work on each problem until MathCheck accepts the solution. Finally he must email the solutions to the teacher in one message. The teacher can copy all solutions from the message to the input box of MathCheck in a single copy-and-paste operation. After clicking the submit button of MathCheck, he sees the solutions in mathematical layout.

The teacher need not check the correctness of each reasoning step, because MathCheck does that. Instead, he can focus on the general flow of the solution. For instance, if he sees $\sin(\frac{\pi}{2}+x)+\sin(\frac{\pi}{2}-x)=\sin\frac{\pi}{2}\cos x+\cos\frac{\pi}{2}\sin x+\sin\frac{\pi}{2}\cos(-x)+\cos\frac{\pi}{2}\sin(-x)=\ldots$, then he may tell the student that the problem can be solved more easily using the formula $\cos x=\sin(x+\frac{\pi}{2})$.

Currently MathCheck can deal with rational functions, absolute values, square root, powers, natural and base-10 logarithms, sin, cos, tan, and derivatives. It checks a relation chain where the relation operators can be $<$, $\leq$, $=$, $>$, and $\geq$. Changing the direction within a chain is not allowed. That is, MathCheck accepts $x^2+2\geq 2>\frac{\pi}{2}$, but refuses to check the last relation of $x^2+2\geq 1<\frac{\pi}{2}$.

MathCheck accepts natural numbers, unsigned mixed numbers, and unsigned decimal numbers. Of course, it accepts $-3\frac{1}{2}$ and $6.023\cdot 10^{23}$, but it interprets them as mathematical expressions instead of single numbers.

Any letters from a to z and A to Z can be used as variables, except e, which denotes the base of the natural logarithm. However, there may be at most three distinct variables in a relation chain. MathCheck interprets i to n and I to N as integer variables and the others as real-valued variables.

MathCheck deems $\frac{x^2}{x}$ different from x, because, if $x = 0$, the former is undefined but the latter yields 0. By writing `#assume x != 0 #enda` to the front of a solution, MathCheck can be made to consider 0 as outside of the domain of x. Then MathCheck accepts $\frac{x^2}{x} = x$. Similarly, as such MathCheck rejects $(\sqrt{x})^2 = x$, but accepts it, if the assumption that $x \geq 0$ has been added to the front. More complicated assumptions can be built with the logical operators $\neg$, $\wedge$, and $\vee$.

When checking a relation, MathCheck first tries to prove the relation true with its rather weak theorem proving ability. If it succeeds, MathCheck draws the relation symbol in green.

Otherwise, it tests the relation by assigning each variable a value, evaluating the expressions on each side of the relation symbol, and comparing the results. It uses precise rational number arithmetic as long as it can, and then switches to double-precision floating points. It repeats the test with different values until it detects a counter-example or runs out of test values. In the latter case, it draws the relation symbol in black. In the case of inequalities, it uses a numeric hill-climbing method to proceed from original test values towards a counter-example.

Antti Rasila (AYO):
Matematiikan koulutusyhteistyöpilotin nykyvaihe ja yhteistyö Abacus-materiaalipoolin kehittämisessä

Esityksessä käydään lyhyesti läpi tekniikan yliopistojen matematiikan e-opetuksen pilottihankkeen [1] toimintaa sekä kerrotaan, miten muut oppilaitokset voivat osallistua hankkeen alulle laittaman materiaalipoolin Abacus [2] kehittämiseen.

Viitteet:

[1] https://abacus.aalto.fi/pilottihanke/

[2] http://abacus.aalto.fi

17 Inverse problems I

Organized by: Petri Ola (HY), Mikko Salo (JY).

Tapio Helin (HY):
Maximum a posteriori estimates in Bayesian inversion

A demanding challenge in Bayesian inversion is to efficiently characterize the posterior distribution. This task is problematic especially in high-dimensional non-Gaussian problems, where the structure of the posterior can be very chaotic and difficult to analyse. Current inverse problem literature often approaches the problem by considering suitable point estimators for the task. Here we discuss the

maximum a posteriori (MAP) estimate and its definition for infinite-dimensional problems. Moreover, we consider how Bregman distance can be used to characterize the MAP estimate. This is joint work with Martin Burger.

Joonas Ilmavirta (JY):
Direct and inverse problems for the p-Laplacian

We consider the PDE $\nabla\cdot(\sigma|\nabla u|^{p-2}\nabla u) = 0$, where the measurable coefficient σ may be zero or infinite in a large set. How does σ being zero or infinite affect existence and uniqueness of solutions? What do the Dirichlet and Neumann boundary values of all solutions to the PDE tell about σ? We present an answer to the first problem and partial answers to the second one. Calderón's inverse problem for this non-linear PDE remains open even for smooth σ.

Sari Lasanen (OY):
PDE's as priors in statistical inverse problemsă

Linear second order elliptic boundary value problemsă with Gaussian white noise loads are applied as priors in statistical inverse problems. For practical calculations, the priors are discretized e.g. with finite element methods.

For Neumann and Robin BVPs the approximating finite-dimensional variational problems are well-defined. However, the usual variational approach to the exact boundary value problem is not applicable, since Gaussian white noise has irregular realizations. In particular, the corresponding Neumann boundary values are not well-defined in the ordinary sense.

In order to guarantee discretization-invariant posterior estimates, the approximationsă are shown to converge. Especially, the limits of finite-dimensional variational problems are studied.ă It turns out that the exact boundary value problem can be formulated by replacing continuity of the boundary trace operator with measurability. The measurability of the boundary trace derives from Cameron-Martin space methods and essentially returns the problem of irregular loads to study of L^2-loads.

18 Inverse problems II

Organized by: Samuli Siltanen (HY).

Paola Elefante (HY):
4D tomography based on a level set method

A novel time-dependent tomographic imaging modality is discussed. The aim is to reconstruct a moving object, such as running engine, a mouse, or a beating human heart, from time-dependent radiographic sparse data The dynamic

three-dimensional structure is reconstructed from projection data using a new computational method. Time is considered as an additional dimension in the problem, and a generalized level set method is applied in space-time. In this approach, the X-ray attenuation coefficient is modeled by the continuous level set function itself (instead of a constant) inside the domain defined by the level set, and by zero outside. Numerical examples with both simulated and measured data suggest that the method successfully regularizes the inverse problem by enforcing continuity both spatially and temporally.

Marko Laine (Finnish Meteorological Institute): Dimension reduction and sampling techniques for inverse problems

Joint work with Tiangang Cui, Zheng Wang (Massachusetts Institute of Technology) and John Bardsley (University of Montana).

In high dimensional and computationally demanding inverse problems, uncertainty quantification (UQ) poses several computational challenges. For Bayesian UQ analysis by sampling from the posterior, the classical methods are based on Markov chain Monte Carlo (MCMC) simulation. In high dimension, the convergence of MCMC in known to be slow, and very large number of forward model simulations are needed.

We present an alternative to MCMC for nonlinear inverse problems that is based on ideas from parametric bootstrap. The randomize-then-optimize (RTO) method computes candidate samples by solving a stochastic optimization problem. In case of a linear forward model, these samples are directly from the posterior density. In the nonlinear case, we derive the form of the sample density and then show how to use it to obtain samples from the posterior distribution of the parameters. For the RTO method, the measurement errors are assumed to be Gaussian, but non-Gaussian prior distributions can be used by suitable non-linear parameter transformations.

In many cases, the dimension of the inverse problem is determined by the discretization of the model state. The intrinsic dimension of the problem can be much lower and it is determined by the degrees of freedom in the measured signal and by the information that comes from the prior. This decomposition allows for efficient dimension reduction techniques. These are used to find the parameter subspaces where the data has information and the subspaces can be efficiently sampled with MCMC or RTO for full Bayesian UQ of the inverse problem.

Both dimension reduction and optimization based sampling are demonstrated using real world applications in atmospheric remote sensing.

Tanja Tarvainen (ISY):
Bayesian approach to quantitative photoacoustic tomography

Photoacoustic tomography (PAT) is an emerging imaging modality developed over the last few decades which combines the benefits of optical contrast and ultrasound propagation. The optical methods provide information about the distribution of chromophores which are light absorbing molecules within the tissue such as haemoglobin, melanin and various contrast agents. The ultrasonic waves carry this optical information directly to the surface with minimal scattering, thus retaining accurate spatial information as well. Nowadays, PAT can be used to provide images of soft biological tissues with high spatial resolution. However, this information is only a qualitative image and it does not include information about the amount of chromophores.

Quantitative photoacoustic tomography (QPAT) is a technique in which the absolute concentration of the chromophores is estimated. This is a hybrid imaging problem in which the solution of one inverse problem acts as a data for another ill-posed inverse problem. In this work, we consider the optical image reconstruction problem of QPAT in Bayesian framework. Results of recent developments using this approach are shown and discussed.

This is joint work with Aki Pulkkinen (University of Eastern Finland), Jari Kaipio (University of Eastern Finland), Ben Cox (University College London), and Simon Arridge (University College London).

19 Logic I, logics, models and classification

Organized by: Åsa Hirvonen (HY).

Kaisa Kangas (HY):
Non-elementary group configuration theorem

Ehud Hrushovski's group configuration theorem is a major source of applications of model theory in other fields of mathematics. It roughly states that whenever there is a certain configuration of elements in a structure, there is a group present. Together with Tapani Hyttinen, I have generalized this theorem to the non-elementary context of quasiminimal classes. I will discuss both Hrushovski's original result and the generalization.

Miguel Moreno (HY):
Borel reducibility and the isomorphism relation

Joint work with T. Hyttinen and V. Kulikov.

There are two ways of classifying first-order theories, one is using Shelah's stability theory and the other one is using descriptive set theory, namely Borel-reducibility and set theoretic complexity of classes of countable models and their

isomorphisms. These two ways, however, do not give the same results: sometimes descriptive set theory gives trivial complexity to theories that are non-trivial in stability theoretic sense and vice versa.

However, if we use generalized descriptive set theory (i.e. studying models of uncountable cardinality), the connection between this and stability theoretic approaches become more obvious. In generalized descriptive set theory, we work in the generalized Baire space κ^κ (κ an uncountable cardinal) and the complexity of a theory is measured by determining the place of the isomorphism relation of models of size κ in the Borel-reducibility hierarchy.

The different kind of theories (classifiable, unstable, strictly stable, etc) have a total or partial characterization so far (see the work of Friedman, Hyttinen, Kulikov, etc.). The connection we study is related to the gap between classifiable and non-classifiable theories; stability theory tells us that classifiable ones are less complex than the non-classifiable ones and also that their complexity are not close (Shelah's Main Gap Theorem).

We prove a Borel-reducibility counterpart of this result employing the equivalence modulo non-stationary ideals. It is consistent that this equivalence relation is with respect to Borel-reducibility strictly above the isomorphism of every classifiable, and strictly below the isomorphism of every non-classifiable theory. It is also consistent that for every classifiable and every non-classifiable theory we can embed the partial order $(P(\kappa), \subset)$ to the Borel-reducibility partial order strictly between the isomorphism relations of these theories.

Jonni Virtema (Leibniz Universität Hannover): Approximation and dependence via multiteam semantics

Abstract

We define a variant of team semantics called *multiteam semantics* based on multisets and study the properties of various logics in this framework. In particular, we define natural probabilistic versions of inclusion and independence atoms and certain approximation operators motivated by approximate dependence atoms of Väänänen.

This is joint work with Arnaud Durand, Miika Hannula, Juha Kontinen, and Arne Meier (for a more detailed exposition see [1]).

Dependence logic was introduced by Väänänen in 2007 [9]. It extends first-order logic with dependence atomic formulas (dependence atoms) $=(\vec{x}, y)$ with the intuitive meaning that the value of the variable y is functionally determined by the values of the variables $\vec{x}$. The notion of dependence has real meaning only in plurals. Thus, in contrast to the usual Tarskian semantics, in dependence logic the satisfaction of formulas is defined not via single assignments but via sets of assignments. Such sets are called *teams* and the semantics is called *team semantics*. We take a further step of replacing structures and teams by their multiset analogues. Multiteams have been considered in some earlier works [6, 7, 10] but so far no systematic study of the subject in the team semantics context has appeared. In the temporal logic setting (in the context of computation tree logic) multiteam

semantics have been introduced and studied recently [8]. In this talk we define the so-called *lax* multiteam semantics and study properties of various logics under this semantics. Moreover we show how the shift from sets to multisets naturally gives rise to probabilistic and approximate versions of dependence logic.

The idea of team semantics goes back to Hodges [5] whose aim was to define compositional semantics for *independence-friendly logic* [4]. The introduction of dependence logic and its many variants has evinced that team semantics is a very interesting and versatile semantical framework. In fact, team semantics has natural propositional, modal, and temporal variants. One of the most important developments in the area of team semantics was the introduction of *independence logic* [3] in which dependence atoms of dependence logic are replaced by *independence atoms* $\vec{x} \perp_{\vec{z}} \vec{y}$. The intuitive meaning of the independence atom $\vec{x} \perp_{\vec{z}} \vec{y}$ is that, when the value of $\vec{z}$ is fixed, knowing the value of $\vec{x}$ does not tell us anything new about the value of $\vec{y}$. Soon after the introduction of independence logic, Galliani [2] showed that independence atoms can be further analysed, and alternatively expressed, in terms of inclusion and exclusion atoms. The inclusion atom $\vec{x} \subseteq \vec{y}$ expresses that each value taken by $\vec{x}$ in a team X appears also as a value of $\vec{y}$ in X. Analogously, the meaning of the exclusion atom $\vec{x}|\vec{y}$ is that $\vec{x}$ and $\vec{y}$ have no common values in X.

A team X over variables $x_1, \ldots, x_n$ can be viewed as a database table with $x_1, \ldots, x_n$ as its attributes. Under this interpretation, dependence, inclusion, exclusion, and independence atoms correspond exactly to functional, inclusion, exclusion, and embedded multivalued dependencies, respectively. These dependencies have been studied extensively in database theory. It is worth noting that multiset semantics (also known as bag semantics) is widely used in databases. On the other hand, independence atoms, embedded multivalued dependencies, and the notion of conditional independence $\vec{X} \perp \vec{Y}|\vec{Z}$ in statistics have very interesting connections. In this talk we establish that, in the multiteam semantics setting, independence atoms can be naturally interpreted exactly as statistical conditional independence.

In practice dependencies such as functional dependence do not hold absolutely but with a small margin of error. In order to logically model such scenarios, Väänänen introduced approximate dependence atoms [10]. In this talk we introduce a general approximation operator which, in particular, can be used to express approximate dependence atoms.

References

[1] Durand, A., Hannula, M., Kontinen, J., Meier, A., Virtema, J.: Approximation and Dependence via Multiteam Semantics. ArXiv e-prints (2015)

[2] Galliani, P.: Inclusion and exclusion dependencies in team semantics - on some logics of imperfect information. Annals of Pure and Applied Logic 163(1), 68–84 (2012)

[3] Grädel, E., Väänänen, J.A.: Dependence and independence. Studia Logica 101(2), 399–410 (2013)

[4] Hintikka, J., Sandu, G.: Informational independence as a semantical phenomenon. In: Logic, Methodology and Philosophy of Science, vol. 8, pp. 571–589. Elsevier, Amsterdam (1989)

[5] Hodges, W.: Compositional semantics for a language of imperfect information. Logic Journal of the IGPL 5(4), 539–563 (electronic) (1997)

[6] Hyttinen, T., Paolini., G., Väänänen, J.: A Logic for Arguing About Probabilities in Measure Teams. ArXiv e-prints (1509.01812) (2015)

[7] Hyttinen, T., Paolini, G., Väänänen, J.: Quantum team logic and bell's inequalities. The Review of Symbolic Logic FirstView, 1–21 (2015)

[8] Krebs, A., Meier, A., Virtema, J.: A team based variant of CTL. In: Proc. TIME 2015 (2015)

[9] Väänänen, J.: Dependence Logic - A New Approach to Independence Friendly Logic, London Mathematical Society student texts, vol. 70. Cambridge University Press (2007)

[10] Väänänen, J.: The Logic of Approximate Dependence. arXiv 1408.4437 (2014)

20 Logic II, computability and games

Organized by: Vesa Halava, Tero Harju (TY).

Lauri Hella (TaY): Inclusion logic and fixed point logic

Joint work with Pietro Galliani.

Inclusion Logic [1], FO($\subseteq$), is a logical formalism designed for expressing inclusion dependencies between variables. It is closely related to Dependence Logic [9], FO(D), which is the extension of First Order Logic FO by functional dependencies between variables. Dependence Logic initially arose as a variant of *Branching Quantifier Logic* [3] and of *Independence-Friendly Logic* [4, 8], and its study has sparked the development of a whole family of logics obtained by adding various dependency atoms to FO.

All these logics are based on Team Semantics [5, 9] which is a generalization of Tarski Semantics. In Team Semantics, formulas are satisfied or not satisfied by *sets* of assignments, called *teams*, rather than by single assignments. This semantics was introduced in [5] for the purpose of defining a compositional equivalent for the Game Theoretic Semantics of Independence-Friendly Logic [4, 8], but it was soon found out to be of independent interest.

Like Branching Quantifier Logic and Independence-Friendly Logic, Dependence Logic has the same expressive power as Existential Second Order Logic Σ^1_1: every FO(D)-sentence is equivalent to some Σ^1_1-sentence, and vice versa [9]. The semantics of Dependence Logic is downwards closed in the sense that if a team X satisfies

a formula ϕ in a model M, then all subteams $Y \subseteq X$ also satisfy ϕ in M. The equivalence between FO(D) and Σ^1_1 was extended to formulas in [7]: FO(D) captures exactly the downwards closed Σ^1_1-definable properties of teams.

On the other hand, it was observed by Galliani [1] that FO($\subseteq$) is strictly weaker than Σ^1_1 and incomparable with FO(D) with respect to formulas. This is simply because the semantics of FO($\subseteq$) is not downwards closed, but is closed under unions: if both teams X and Y satisfy a formula ϕ in a model M, then $X \cup Y$ also satisfies ϕ in M. Moreover, Galliani proved that FO($\subseteq$) is stronger than FO over sentences, and it is contained in Σ^1_1.

In the article [2], we gave a complete characterization for the expressive power of FO($\subseteq$) in terms of Positive Greatest Fixed Point Logic GFP$^+$: every FO($\subseteq$)-sentence is equivalent to some GFP$^+$-sentence, and vice versa. Fixed point logics have a central role in Descriptive Complexity Theory. By the famous result of Immerman [6] and Vardi [10], Least Fixed Point Logic LFP captures PTIME on the class of ordered finite models. Furthermore, it is well known that on finite models, LFP is equivalent to GFP$^+$. Thus, we obtained a new characterization for PTIME: a class of ordered finite models is in PTIME if and only if it is definable in FO($\subseteq$).

Furthermore, we proved in [2] that all union-closed first-order definable properties of teams are definable in Inclusion Logic Thus, it is not possible to increase the expressive power of FO($\subseteq$) by adding first-order definable union-closed dependencies. On the other hand, it is an interesting open problem, whether FO($\subseteq$) can be extended by some natural set $\mathbf{D}$ of union-closed dependencies such that the extension FO($\subseteq, \mathbf{D}$) captures all union-closed Σ^1_1-definable properties of teams.

We also introduced a new Ehrenfeucht-Fraïssé game that characterizes the expressive power of Inclusion Logic. Our game is a modification of the EF game for Dependence Logic defined in [9]. Although the EF game has a clear second order flavour, it is still more manageable than the usual EF game for Σ^1_1; we illustrate this by describing a concrete winning strategy for Duplicator in the case of models with empty signature.

In the talk, I will first explain the basic concepts pertaining team semantics and extensions of FO with various dependencies. Then I will give a survey on the results in the paper [2].

References

[1] P. Galliani. Inclusion and exclusion dependencies in team semantics: On some logics of imperfect information. *APAL*, 163(1):68 – 84, 2012.

[2] P. Galliani and L. Hella. Inclusion logic and fixed point logic. In *Computer Science Logic 2013 (CSL 2013)*, pages 281–295, 2013.

[3] L. Henkin. Some Remarks on Infinitely Long Formulas. In *Infinitistic Methods. Proc. Symposium on Foundations of Mathematics*, pages 167–183. Pergamon Press, 1961.

[4] J. Hintikka and G. Sandu. Informational independence as a semantic phenomenon. In J. Fenstad, I. Frolov, and R. Hilpinen, editors, *Logic, methodology and philosophy of science*, pages 571–589. Elsevier, 1989.

[5] W. Hodges. Compositional Semantics for a Language of Imperfect Information. *Journal of the Interest Group in Pure and Applied Logics*, 5 (4):539–563, 1997.

[6] N. Immerman. Relational queries computable in polynomial time. *Information and control*, 68(1):86–104, 1986.

[7] J. Kontinen and J. Väänänen. On definability in dependence logic. *Journal of Logic, Language and Information*, 3(18):317–332, 2009.

[8] A. L. Mann, G. Sandu, and M. Sevenster. *Independence-Friendly Logic: A Game-Theoretic Approach*. Cambridge University Press, 2011.

[9] J. Väänänen. *Dependence Logic*. Cambridge University Press, 2007.

[10] M. Y. Vardi. The complexity of relational query languages. In *Proceedings of the fourteenth annual ACM symposium on Theory of computing*, pages 137–146. ACM, 1982.

Kerkko Luosto (TaY): Set systems combinatorics related to modal logics

We study modal logics in the framework of Kripke structures, but instead of normal pointed semantics, we use team semantics introduced (in this context) by Jouko Väänänen. Then the formulas of modal logic give rise to truth-families of sets instead of truth-sets, i.e.,

$$\|\phi\|^{\mathcal{K}} = \{T \mid \mathcal{K}, T \models \phi\}$$

is a family of sets. When the standard modal logics is enhanced with either so-called *dependence atoms* or *intuitionistic disjunction*, the structures of these families become interesting. We show how to create appropriate dimension concepts to analyze the expressive power of the modal logics under consideration.

This is joint work with Lauri Hella, Katsuhiko Sano and Jonni Virtema

Reino Niskanen (University of Liverpool): Equivalent reachability games with contrasting rules

The standard reachability games for two player are zero-sum games played on a directed graph where the vertices are partitioned between the two players, commonly called Adam and Eve. They move a token on the graph with Eve's goal to reach a certain target vertex while Adam wants to keep Eve from doing that. In other words, Eve has a reachability objective and Adam has a safety objective (i.e., avoiding Eve's objective). One of decision problems for these games is whether

Eve has a winning strategy, i.e., can she eventually reach the target regardless of the moves chosen by Adam.

Reachability games provide a powerful mathematical framework for a large number of computational problems. In particular they appear in the verification, refinement, and compatibility checking of reactive systems, analysis of programs with recursion, combinatorial topology and have deep connections with automata theory and logic. In many cases the most challenging problems appear in low-dimensional models or systems. These systems do not have enough space to encode a universal computation (i.e., that with the computational power of a Turing machine) in a direct way and at the same time demonstrate complex and unpredictable behaviour.

This talk is focusing on several variants of infinite-state reachability games. We consider a particular class of reachability games where moves of players are associated with vectors in $\mathbb{Z}^n$. These are called *counter reachability games*. Now, apart from reaching the target state, Eve's goal is, at the same time, to reach it with a certain counter value. We consider different variants of counter reachability games that seem to be essentially different but turn out to be mutually reducible. We show some logical tricks that are commonly used in the proofs. A lot of them exploit the fact that the players have the opposite objectives and utilize different gadgets that allow one player to ensure his/her victory if the other player dos not play in a certain way.

21 Mathematical physics

Organized by: Mikko Stenlund (HY).

Christian Webb (AYO): Global fluctuations of the spectrum of large random matrices

We consider different objects encoding the global behavior of the eigenvalues of some large random matrices and discuss how logarithmically correlated Gaussian fields arise from these objects as the size of the matrix increases. In particular, we discuss how this ties into a central limit theorem for the so-called linear statistics of the matrices and how such central limit theorems can be proven.

Eveliina Peltola (HY): Quantum groups and random geometry

Critical lattice models of statistical mechanics have been intensively studied in the past decades. At the scaling limit, these models are expected to converge to quantum field theories with a powerful symmetry, conformal invariance – conformal field theories (CFT). Mathematical results about the conformal invariance of the scaling limits have been recently established, but only in some cases. Surprisingly, in CFT there is a hidden symmetry, implemented by quantum groups. In this talk, I describe how quantum group methods can be used to solve problems in random

geometry and CFT. I present applications to Schramm-Loewner evolutions, which emerge as (conjectured) scaling limits of interfaces in lattice models.

The talk is based on joint work with Kalle Kytölä and Steven Flores.

Tuomas Sahlsten (University of Bristol): Quantum ergodicity on hyperbolic surfaces with large injectivity radius

Quantum chaos is a field in mathematical physics based on the Bohr correspondence principle stating that the high energy behaviour of quantum observables should be related to the behaviour of the underlying classical dynamical system. A classical result in this field is the Quantum Ergodicity Theorem of A. I. Šnirel'man, Y. Colin de Verdière and S. Zelditch which states that on any compact Riemannian manifold M, when the geodesic flow is ergodic with respect to the Liouville measure, then for any orthonormal basis $\{\psi_j\}$ of eigenfunctions of the Laplacian on M, we can find a subsequence of density 1 of the measures

$$|\psi_j|^2 \, d\mathrm{Vol}_M$$

that converges to the volume measure $d\mathrm{Vol}_M$ on M in the large eigenvalue limit (i.e. when the energy increases to infinity).

In our work, instead of taking large eigenvalue limits, we study what happens if we just fix a spectral window and change geometric parameters of M. In particular, we will study what happens on constant negatively curved manifolds M if we let the injectivity radius increase. Our approach is based on recent ideas of Brooks, Lindenstrauss and Le Masson (IMRN, 2015) on quantum ergodicity on large regular graphs (which was earlier developed by Anantharaman and Le Masson (Duke Math. J., 2015)), and is the 'continuous' analogue of these ideas. The talk should provide a gentle introduction to quantum chaos and to our ongoing work.

This is joint work with Etienne Le Masson (Bristol).

22 Mathematical systems theory

Organized by: Seppo Pohjolainen (TTY).

Jukka-Pekka Humaloja (TTY): Robust regulation for first-order port-Hamiltonian systems

Joint work with Lassi Paunonen and Seppo Pohjolainen.

We present a method for obtaining robust control over a first-order port-Hamiltonian system. The presented method is especially designed for controlling impedance energy-preserving port-Hamiltonian systems. A linear first-order port-

Hamiltonian system on the spatial interval $\zeta \in [a,b]$ is given by

$$\frac{\partial}{\partial t}x(\zeta,t) = \mathcal{A}x(\zeta,t), \quad x(0) = x_0, \tag{10a}$$

$$u(t) = \mathcal{B}x(\zeta,t), \tag{10b}$$

$$y(t) = \mathcal{C}x(\zeta,t), \tag{10c}$$

where $\mathcal{B}$ and $\mathcal{C}$ are bounded linear operators, and the operator $\mathcal{A}$ is defined by

$$\mathcal{A}x(\zeta,t) := P_1\frac{\partial}{\partial\zeta}(\mathcal{H}(\zeta)x(\zeta,t)) + P_0\mathcal{H}(\zeta)x(\zeta,t), \tag{11}$$

where $P_1 \in \mathbb{C}^{n\times n}$ is invertible and self-adjoint $P_0 \in \mathbb{C}^{n\times n}$ is skew-adjoint, and $\mathcal{H}(\zeta) \in C^1([a,b];\mathbb{C}^{n\times n})$ such that $\mathcal{H}(\zeta)$ is self-adjoint for all $\zeta \in [a,b]$, and there exists $M, m > 0$ such that $mI \leq \mathcal{H}(\zeta) \leq MI$ for all $\zeta \in [a,b]$. An impedance energy-preserving systems satisfies the relation

$$\frac{1}{2}\frac{d}{dt}||x(\zeta,t)||_X^2 = u^*(t)y(t), \tag{12}$$

where the norm $||x(\zeta,t)||_X^2$ in the state-space $X = L^2([a,b];\mathbb{C}^n)$ is defined as

$$||x(\zeta,t)||_X^2 = \langle x(\zeta,t), x(\zeta,t)\rangle_X = \frac{1}{2}\int_a^b x(\zeta,t)^*\mathcal{H}(\zeta)x(\zeta,t)d\zeta. \tag{13}$$

We will show that a controller of the form

$$\dot{z}(t) = \mathcal{G}_1 z(t) + \mathcal{G}_2\big(y(t) - y_{ref}(t)\big), \qquad z(0) = z_0, \tag{14a}$$

$$u(t) = Kz(t) - \kappa y(t), \tag{14b}$$

where y_{ref} is a reference signal generated by an exosystem S, with suitably chosen parameters $(\mathcal{G}_1, \mathcal{G}_2, K, \kappa)$ achieves robust output regulation for the initially unstable system (10a)–(10c) satisfying relation (12). An example will be given where we implement a robust regulating controller for one-dimensional wave equation with boundary control and observation.

Mikael Kurula (ÅA):
De Branges-Rovnyak functional models for operator Schur functions on $\mathbb{C}^+$

Joint work with Joseph A. Ball and Olof J. Staffans.

Let $\mathcal{U}$ and $\mathcal{Y}$ be separable Hilbert spaces and let φ be an associated *Schur function*, i.e., φ is analytic on the open complex right-half plane with values that are contractions from $\mathcal{U}$ into $\mathcal{Y}$. Then the kernel function

$$K_o(\mu,\lambda) = \frac{1-\varphi(\mu)\varphi(\lambda)^*}{\mu+\overline{\lambda}}, \qquad \mu,\lambda \in \mathbb{C}^+,$$

is positive; hence it is the reproducing kernel of a Hilbert space $\mathcal{H}_o$. Using this as state space, we have the following realization result:

Theorem 1. *For an arbitrary Schur function on $\mathbb{C}$, define $\mathcal{H}_o$ as above.*

1. *The following unbounded operator $\left[\begin{smallmatrix}\mathcal{H}_o\\ \mathcal{U}\end{smallmatrix}\right] \to \left[\begin{smallmatrix}\mathcal{H}_o\\ \mathcal{Y}\end{smallmatrix}\right]$ is an observable, co-energy-preserving system node with state space $\mathcal{H}_o$, input space $\mathcal{U}$, and ouput space $\mathcal{Y}$:*

$$\begin{bmatrix} A\&B \\ C\&D \end{bmatrix}_o : \begin{bmatrix} x \\ u \end{bmatrix} \mapsto \begin{bmatrix} z \\ y \end{bmatrix}, \quad \textit{where}$$

$$z(\mu) := \mu x(\mu) + \varphi(\mu)u - y, \qquad \mu \in \mathbb{C}, \tag{15}$$

$$y := \lim_{\mathrm{Re}\,\eta \to +\infty} \eta\, x(\eta) + \varphi(\eta)u, \tag{16}$$

$$\mathrm{dom}\left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]_o\right) = \left\{ \begin{bmatrix} x \\ u \end{bmatrix} \in \begin{bmatrix} \mathcal{H}_o \\ \mathcal{U} \end{bmatrix} \;\middle|\; \exists y \in \mathcal{Y} :\ z \text{ in } (15) \text{ lies in } \mathcal{H}_o \right\}.$$

For every $\left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right] \in \mathrm{dom}\left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]_o\right)$, the $y \in \mathcal{Y}$ such that z given in (15) is in $\mathcal{H}_o$, is unique and it is given by (16).

2. *The transfer function of $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]_o$ is equal to φ on $\mathbb{C}$:*

$$C_o\&D_o \begin{bmatrix} (\lambda - A_o|_{\mathcal{H}_o})^{-1} B_o \\ I_{\mathcal{U}} \end{bmatrix} = \varphi(\lambda), \qquad \lambda \in \mathbb{C}.$$

The preceding result is a right-half-plane analogue of the closely-outer-connected, co-isometric de Branges-Rovnyak functional model for a Schur function on the complex unit disk [1, 2]. We also have the following uniqueness result which says that the observable co-energy-preserving realization of φ is unique up to unitary equivalence:

Theorem 2. *Let $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]$ be an arbitrary observable and co-energy-preserving realization of φ with state space $\mathcal{X}$. The mapping*

$$\Delta : K_o(C\&Dot, \lambda)y \mapsto \left(\overline{\lambda} - A^*|_{\mathcal{X}}\right)^{-1} C^* y, \qquad \lambda \in \mathbb{C}^+,\ y \in \mathcal{Y},$$

extends by linearity and continuity into a unitary operator from $\mathcal{H}_o$ onto $\mathcal{X}$. The operator $\left[\begin{smallmatrix} \Delta & 0 \\ 0 & 1_{\mathcal{U}} \end{smallmatrix}\right]$ maps $\mathrm{dom}\left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]_o\right)$ one-to-one onto $\mathrm{dom}\left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]\right)$, and

$$\begin{bmatrix} A\&B \\ C\&D \end{bmatrix} \begin{bmatrix} \Delta & 0 \\ 0 & 1_{\mathcal{U}} \end{bmatrix} = \begin{bmatrix} \Delta & 0 \\ 0 & 1_{\mathcal{Y}} \end{bmatrix} \begin{bmatrix} A\&B \\ C\&D \end{bmatrix}_o.$$

It is a common approach to use a linear fractional transformation to transfer results from the disk case into the right-half-plane case, but we avoid this in order to expose the essential technical differences to the disk setting (in the full papers), such as implications of the unboundedness of most of the involved operators.

In addition to the above realization, we give a controllable energy-preserving and a simple conservative de Branges-Rovnyak functional model on $\mathbb{C}^+$. The presentation starts with a recollection of the necessary background from continuous-time, infinite-dimensional systems theory. We refer to [3, 4] for details and proofs.

References

[1] L. de Branges and J. Rovnyak. Canonical models in quantum scattering theory. In *Perturbation Theory and its Applications in Quantum Mechanics*, pages 295–392. Wiley, New York, 1966.

[2] L. de Branges and J. Rovnyak. *Square summable power series*. Holt, Rinehart and Winston, New York, 1966.

[3] J. A. Ball, M. Kurula, O. J. Staffans, and H. Zwart. De Branges–Rovnyak Realizations of Operator-Valued Schur Functions on the Complex Right Half-Plane. *Complex Anal. Oper. Theory*, 9(4):723–792, 2015.

[4] J. A. Ball, M. Kurula, and O. J. Staffans. A conservative de Branges-Rovnyak functional model for operator Schur functions on $\mathbb{C}$, manuscript, 2016.

Jarmo Malinen (AYO): Webster's equation in acoustic waveguides

Long wavelength acoustics in lossy, curved tubular waveguides can be approximated by the generalised Webster's model, consisting of the PDE

$$\frac{1}{c^2\Sigma(s)^2}\frac{\partial^2\psi}{\partial t^2} = \frac{1}{A(s)}\frac{\partial}{\partial s}\left(A(s)\frac{\partial\psi}{\partial s}\right) - \frac{2\pi\alpha W(s)}{A(s)}\frac{\partial\psi}{\partial t} \quad \text{for } s\in[0,\ell], t\geq 0,$$

together with the appropriate boundary conditions at $s=0,\ell$. Here A represents the spatially varying intersectional area of the waveguide, the function Σ is a correction due to curvature, and the function W in the dissipation term is due to different inner and outer curvature radii of the waveguide. The solution $\psi=\psi(s,t)$ is the velocity potential, and the acoustic pressure as well as the (perturbation) velocity can be obtained from it by partial derivatives.

Two problems related to Webster's model are considered, and their solutions are presented in the special case $\alpha=0$ and $\Sigma(s)=1$. Firstly, the control $A(0)\phi_s(0,t)=i(t)$ and observation $p(t)=\rho\phi_t(\ell,t)$ are used, and it is shown that the impedance transfer function from i to p does not have zeroes if A is real analytic in a neighbourhood of ℓ.

Secondly, first order perturbations are derived for the eigenvalues λ^2 of the resonance problem

$$\left(\frac{\lambda}{c}\right)^2\Psi = \frac{1}{A(s)}\frac{\partial}{\partial s}\left(A(s)\frac{\partial\Psi}{\partial s}\right) \text{ for } s\in[0,\ell]$$

for small perturbations $A=A_0+\epsilon B$ of a nominal area function A_0. As an application, local spectral inversion problem is considered. Given (measurement) data about the some of the eigenvalues λ_j^2 relating to a true area function A (of which we only have a guess, i.e., $A_0\approx A$), how can be find a first order correction B to A_0?

As a practical application of both the problems, modelling of speech acoustics of vowels is discussed.

23 Number theory

Organized by: Kaisa Matomäki (TY).

Jukka Keränen (AYO): Compact support cohomology of Picard modular surfaces

We compute the cohomology with compact supports of a Picard modular surface as a virtual module over the product of the appropriate Galois group and the appropriate Hecke algebra. We use the method developed by Ihara, Langlands, and Kottwitz: comparison of the Grothendieck–Lefschetz formula and the Arthur–Selberg trace formula. Our implementation of this method takes as its starting point the recent work of Laumon and Morel. The principal application of this computation is to express the Hasse–Weil L-function of a Picard modular surface as a product of automorphic L-functions. As such, our work belongs to the Langlands program.

Mika Mattila (TTY), Jori Mäntysalo (TaY): Studying the Bourque-Ligh conjecture with and without program Sage

Joint work with Pentti Haukkanen (TaY).

Let $S = \{x_1, x_2, \ldots, x_n\}$ be a finite set of positive integers with $x_1 < x_2 < \cdots < x_n$. The GCD and LCM matrices of the set S are the $n \times n$ matrices with $\gcd(x_i, x_j)$ and $\operatorname{lcm}(x_i, x_j)$ as their ij entries, respectively. In 1992 Bourque and Ligh [1] presented the famous conjecture that if the set S is GCD closed (that is, $\gcd(x_i, x_j) \in S$ for all $i, j \in \{1, 2, \ldots, n\}$), then the LCM matrix of the set S is invertible. Haukkanen et al. [3] soon showed that this conjecture does not hold in general, and subsequently it was shown by Hong [4] that the conjecture holds if there are at most 7 elements in the set S, but the conjecture does not hold in general for larger sets.

Hong's number-theoretic proof required an extensive case-study, and the same can be said about another, more recent proof by Korkee et al. [5]. In this new approach the key idea is that instead of thinking of S as a subset of $\mathbb{Z}^+$, we may focus on the structure of $(S, |)$ and on the Möbius function of the poset $(S, |)$. By going through all possible finite semilattice structures with at most 7 elements Korkee et al. are able to find an alternative proof for Bourque-Ligh conjecture. This same lattice-theoretic approach was also utilized in [2].

Although there is nothing left to prove in the Bourque-Ligh conjecture itself, there are still many open questions that are related to the conjecture. Why does the conjecture fail when there are exactly 8 elements in S but not earlier or later? How is it possible that in many cases the semilattice structure of S alone suffices to guarantee the invertibility of the matrix $[S]$? In this presentation we are going to continue our lattice-theoretic study and we are able to answer these questions.

Last we will demonstrate how the program Sage can be used to help in the lattice-theoretic study of the Bourque-Ligh conjecture. It turns out that the program can contribute in many ways such as constructing all semilattices of given size, testing hypotheses and finding counterexamples.

References:

[1] K. Bourque and S. Ligh, On GCD and LCM matrices, *Linear Algebra Appl.* 174 (1992) 65–74.

[2] P. Haukkanen, M. Mattila and J. Mäntysalo, Studying the singularity of LCM-type matrices via semilattice structures and their Möbius functions, *J. Combin. Theory Ser. A* 135 (2015) 181–200.

[3] P. Haukkanen, J. Wang and J. Sillanpää, On SmithŠs determinant, *Linear Algebra Appl.* 258 (1997) 251–269 .

[4] S. Hong, On the Bourque-Ligh conjecture of least common multiple matrices, *J. Algebra* 218 (1999) 216–228.

[5] I. Korkee, P. Haukkanen and M. Mattila, A lattice-theoretic approach to the Bourque-Ligh conjecture, submitted to *Discrete Math.*

Esa V. Vesalainen (AYO): Sums involving Fourier coefficients of $GL(n)$ Maass forms

We first introduce the important class of automorphic forms called $GL(n)$ Maass forms, and then describe some pointwise, moment, resonance and omega estimates for sums involving their Fourier coefficients. (Joint work with A.-M. Ernvall-Hytönen and J. Jääsaari.)

24 Statistics

Organized by: Hannu Oja (TY).

Pauliina Ilmonen (AYO): But how rare is rare?

Modeling extreme events is of paramount importance in various areas of science – biostatistics, climatology, finance, geology, and telecommunications, to name a few. Since these real world phenomena are often in dimension higher than one, development of estimators for multivariate tail behavior is desired. We consider estimating the tail behavior of certain heavy-tailed multivariate distributions. In other words, in multivariate settings, we try to answer to the question: "How rare is rare?"

Esa Ollila (AYO): Regularized discriminant analysis

We consider the problem of jointly estimating unknown covariances of K datasets, each containing n_k samples, under the assumption that the true covariance matrices

are close to each other in some specific metric. Motivated by the discriminant analysis applications in the undersampled scenario, we assume that the covariances possess a positive definite mutual center, which is jointly estimated from the given data. We use a penalized M-estimation approach in which we maximize the sum of the individual M-estimation cost functions of the samples with an additive joint penalty enforcing similarity with (shrinkage towards) the joint center. We utilize the concept of geodesic convexity and prove the existence and uniqueness of the penalized solution under general conditions. We use three different penalty functions based on Riemannian, information theoretic (Kullback-Leibler) distances as well as an ellipticity measure of positive definite matrices. These are shown to lead to joint center estimates that are related to Riemannian and harmonic means of positive definite matrices, respectively. Due to technical details, Tyler's M-estimator requires separate treatment, which we also provide in the paper. Fixed point equations are derived for each penalty function and the benefits of different estimators are illustrated by numerical simulations and a real data example. This is a joint work with Ami Wiesel, Ilya Soloveychik and David E. Tyler

Mikko J. Sillanpää (OY): An efficient genome-wide multilocus epistasis search

High-throughput laboratory techniques are producing vast amount of genomic marker data - discrete predictors to association studies. Linear regression model is often considered to link study phenotypes and these marker measurements to each other. Number of predictors in multi-marker regression models can easily be much larger than number of observations. Therefore, one needs application of variable selection to find small subset of important predictors out of large number of candidates. Such models can occasionally include also all pairwise locus-by-locus (epistasis) interactions which increases dimensionality of the model very rapidly.

We consider variable selection problem of linear model containing large amount of predictors and all of their pairwise interactions in the model jointly. Our suggested approach (Kärkkäinen et al. 2015) use sure-independence-screening to first drop dimension of the problem by considering marginal importance of each interaction term within the huge loop. Subsequent estimation step then consider Bayesian variable selection approach (Extended Bayesian LASSO - Mutshinda and Sillanpää 2010). We also show that it is important to separate search of main and interaction effects in the algorithm to control number of false positives. Examples illustrates superior performance of our method over PLINK in terms of computation time and empirical power. Our successful examples consider even problem of originally of order of 280,000,000 interactions within a reasonable time frame.

REFERENCES

Mutshinda CM, Sillanpää MJ (2010) Extended Bayesian LASSO for multiple quantitative trait loci mapping and unobserved phenotype prediction. Genetics 186: 1067-1075.

Kärkkäinen HP, Li Z, Sillanpää MJ (2015) An efficient genome-wide multilocus epistasis search. Genetics 201: 865-870.

25 Stochastic I, stochastic analysis and related topics

Organized by: Stefan Geiss (JY), Paavo Salminen (ÅA).

Andreas Anckar (ÅA): Random compositions of Fibonacci and Padovan recurrences

Joint work with Göran Högnäs.

We study the random integer sequence given by the recurrence relation

$$x_{n+3} = x_{n+1+\xi} + x_{n+\xi},\ x_0 = x_1 = x_2 = 1$$

where ξ equals 0 or 1 with equal probability. That is, at every step we choose the next element according to either the Fibonacci or the Padovan recurrence relation. The main focus is on calculating the growth rate

$$\gamma = \lim_{n\to\infty} \sqrt[n]{x_n},$$

which exists almost always. The problem is inspired by a classic similar treatise on random Fibonacci sequences ([1]) and a computation of the constant $\lim_{n\to\infty} \sqrt[n]{\mathbb{E}\left(x_n\right)}$ in our case ([2]).

Let $F(x,y,z) = (y,z,y+z)$ and $P(x,y,z) = (y,z,x+y)$ and consider the Markov chain $X_n = M\left(X_{n-1}\right)$ where M equals P or F with equal probability, and $X_0 = (1,1,1)$. Then we can write $X_n = |X_n|\,\overline{X}_{n-1}$ where $|(x,y,z)| = x+y+z$ and $\overline{(x,y,z)} = \frac{1}{x+y+z}\,(x,y,z)$. If we denote the distribution of $\overline{X}_n$ by π, we have

$$\log\gamma = \frac{1}{2}\int \left(\log|Fx| + \log|Px|\right) d\pi(x).$$

The chain $\overline{X}_n$ is uniformly ergodic and thus we can find ([3], section 16.2) $\rho > 0$ such that

$$\|\mathbb{P}^n(x,\cdot) - \pi\| \leq \rho^n.$$

Our aim is to use these methods to approximate π (and consequently γ) as well as possible.

References

[1] Viswanath, D.: Random Fibonacci sequences and the number 1.13198824, Mathematics of Computation 69 (231), 1131-1156, 1999

[2] Gogin, N. and Mylläri, A.: On the Average Growth Rate of Random Compositions of Fibonacci and Padovan Recurrences, Computer Algebra in Scientific Computing, Lecture Notes in Computer Science, Volume 5743, 2009

[3] Meyn, S.P. and Tweedie, R.L.: Markov chains and stochastic stability. Springer-Verlag, London, 1993

Antti Luoto (JY):
On the mean first exit time of a Brownian bridge with drift

The mean first exit time $\mathbb{E}_0\left[\tau_{(-h,h)}\right]$ of the standard Brownian motion exiting the interval $(-h,h)$ is known to be h^2. In my talk I present a new representation of the mean first exit time $\mathbb{E}_{0,T,y}\left[\tau_{(-h,h)}\right]$ of a Brownian bridge moving from 0 to a point $y \notin (-h,h)$ within the time interval $[0,T]$. This representation allows to conclude the asymptotic behavior

$$\lim_{h\to 0} \frac{\mathbb{E}_{0,T,y}\left[\tau_{(-h,h)}\right]}{\mathbb{E}_0\left[\tau_{(-h,h)}\right]} = 1.$$

The behavior of $\mathbb{E}_{0,T,y}\left[\tau_{(-h,h)}\right]$ as a function of the end point y is also discussed.

Lauri Viitasaari:
Some developments of pathwise (stochastic) integration theory

In this talk some new results on pathwise integrals are discussed. In particular, results focus on cases where the integrand or integrator are of unbounded p-variation for any $p \geq 1$ and consequently, the existing methodology to define pathwise integrals including rough path analysis cannot be applied. The approach is based on generalised Lebesgue–Stieltjes integrals and fractional calculus.

26 Stochastic II, stochastic processes and applications

Organized by: Luis Alvarez Esteban (TY).

Jaakko Lehtomaa (HY):
Large deviations of means of heavy-tailed random variables with finite moments of all orders

Logarithmic asymptotics of the mean process $\{S_n/n\}$ are investigated in the presence of heavy-tailed increments. As a consequence, full large deviations principle for means is obtained when the hazard function of an increment is regularly varying with index $\alpha \in (0,1)$. This class includes all stretched exponential distributions. Thus the previous research of Gantert et al is extended. Furthermore, the presented proofs are more transparent than the techniques used by Nagaev et al. In addition, the novel approach is compatible with other common classes of distributions, e.g. those of lognormal-type.

Lasse Leskelä (AYO):
Directed random intersection graphs

Unlike most classical random graph models studied in the literature, many real-world social and data networks are directed, and neighboring nodes are often statistically correlated. In this talk I will discuss a mathematical model of a large directed graph where stochastic dependence between neighboring nodes is built into the model by randomly assigining each node two sets of attributes. An ordered node pair is connected if one node demands an attribute that the other node supplies. In this talk I will describe conditions on the model parameters for the existence of a giant strongly connected component, and results describing the expected size of the forward and backward components of a typical node in the network. The talk is based on joint work with Mikko Kuronen (University of Jyväskylä, Finland).

Adil Yazigi (VY):
Conditional full support for multivariate brownian moving average

We investigate when multivariate Brownian moving average processes have conditional full support (CFS): a non-degenerate property which informally means that after any time, the process still has the possibility to go anywhere. Based on Cherny's work (2008) where the CFS property has been shown for univariate Brownian moving average processes, we provide a multivariate generalization of Cherny's result and show that the CFS holds if the convolution determinant condition is satisfied.