

**International Workshop
on
QUASICONFORMAL MAPPINGS AND THEIR APPLICATIONS**

December 27, 2005 - January 01, 2006

Indian Institute of Technology Madras
Department of Mathematics

Proceedings of the
International Workshop
on
Quasiconformal Mappings And Their Applications
(IWQCMA05)
December 27, 2005 - January 01, 2006



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Mathematical Union**
- **Indian National Science Academy, India**
- **Council of Scientific and Industrial Research, India**
- **Department of Science and Technology, India**

Preface

The Department of Mathematics, IIT Madras, Chennai hosted International Workshop on Quasiconformal Mappings and their Applications (IWQCMA05) December 27, 2005- January 1, 2006. This event was the first one in India on this active research area which has its roots in geometric function theory and which is closely connected with several topics of mathematical analysis.

The organizers gratefully acknowledge the financial support of

1. National Board for Higher Mathematics (DAE), India
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for their support. Many people have given us help, in particular the students of Prof. S.Ponnusamy. Prior to the start of the workshop, preworkshop lectures were given by Dr. Antti Rasila and Prof. Raimo Näkki from Finland. We thank both of them. Also, we take this opportunity to thank Prof. R. Balasubramanian, Prof. R. Parvatham, and Prof. C. S. Seshadri for their continued encouragement and helpful advice.

The participants, who represented many different countries, received in most cases financial support from their national funding organizations to cover their expenses. ICTP's generous support was useful in supporting mathematicians from developing countries. Without the invaluable support from the aforementioned organizations, this conference would not have been possible in its present form.

The main goal of the conference was to bring together internationally well-known experts representing geometric function theory and some related topics. They were requested to deliver a series of lectures for postgraduate students on their respective areas. The audience consisted of mathematicians ranging from graduate students to well-known experts from all the participating countries.

Conformal invariance and conformally invariant metrics have been important research topics in geometric function theory during the past century. These topics also were discussed or mentioned in several of the lectures. The organizing committee was pleased to observe that the lectures were very well received and

lead to many lively discussions afterwards. We were also pleased to receive positive response from the speakers to our request to contribute their lectures for the proceedings. It is our hope that the publication of these proceedings will make the results presented in this Workshop and also this research area and its challenging open problems more widely known for a wide readership than what is the case presently. The editorial work was carried out at IIT Madras, and the www-pages

<http://mat.iitm.ac.in/samy/>

<http://www.cajpn.org/madras/>

<http://www.math.utu.fi/proceedings/madras>

contain a copy of these proceedings.

Special thanks to Mr. Swadesh Kumar Sahoo, and Mrs. P. Vasundhra for their help in the organization of the meeting. We take this opportunity to thank Prof. Roger W. Barnard for his support in getting NSF grant for supporting US participants, and for its partial support in bringing out this proceedings.

On behalf of the Organizing committee

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